

University of Saskatchewan  
Department of Computer Science

**Numerical Software**  
(CMPT 898 (02))

Instructor: Dr. Raymond J. Spiteri

MILESTONE 02

**Closes: 2:30 p.m., Thursday, February 14, 2013**

1. **[50 marks]** Go to

[http://software-carpentry.org/4\\_0/index.html](http://software-carpentry.org/4_0/index.html)

Watch the set of videos associated with your assigned topic (see below).

Student	Video Series
Frost	Program Design
Haghgoo	Version Control
Mirshekari	Classes and Objects
Preuss	Systems Programming
Qian	Testing

Prepare a brief presentation (5–10 mins.) on your assigned topic. The presentations should provide a summary of the three most important points made in the videos. Emphasis is to be placed on identifying what you learned or found particularly interesting or useful. The presentations will take place Tuesday, February 05, 2013 during class, with the order of the presenters to be determined. Slides are to be e-mailed in PDF format to the instructor by noon of the presentation day.

Everyone is encouraged to watch all the assigned videos so they can ask questions or add insight to the presentations.

2. **[10 marks]** On a certain computer, the distance between 7 and the next larger floating-point number is  $2^{-12}$ . What is machine epsilon on that computer? What is the distance between 70 and the next larger floating-point number on that computer? Assume of course that the computer represents numbers in base 2.
3. **[10 marks]** Write down the roots of the polynomial:  $W(x) = (x-1)(x-2) \dots (x-20)$ . Now suppose that we perturb one of the coefficients by a very small amount: define a new polynomial

$$p(x) = W(x) - 2^{-23}x^{19}.$$

Find the all the roots of this new polynomial using MATLAB.

Is the problem of finding the roots of  $W(x)$  sensitive? Explain.

4. [30 marks] In this question we will examine the stability of computing the factors of the *singular value decomposition*.

The singular value decomposition (SVD) of a matrix  $\mathbf{A} \in \mathbb{R}^{m \times n}$  is a factorization of the form

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T,$$

where  $\mathbf{U} \in \mathbb{R}^{m \times m}$  and  $\mathbf{V} \in \mathbb{R}^{n \times n}$  are orthogonal, and  $\mathbf{\Sigma} \in \mathbb{R}^{m \times n}$  is a “diagonal” matrix with elements  $\sigma_{11} \geq \sigma_{22} \geq \dots \geq \sigma_{nn} \geq 0$  and all other elements 0.

- (a) Name 3 practical applications of the SVD.
  - (b) In MATLAB, we construct a random matrix  $\mathbf{A} \in \mathbb{R}^{50 \times 50}$ , the SVD of which we know exactly, as follows: construct two random orthogonal matrices  $\mathbf{U}$  and  $\mathbf{V}$  from two normally random matrices using the **QR** decomposition and a diagonal matrix  $\mathbf{S}$  with (ordered) elements from a uniform random distribution on  $[0, 1]$ . Set  $\mathbf{A} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}'$ .
  - (c) Find the SVD of  $\mathbf{A}$ ; call the computed factors  $\mathbf{U}_{\text{tilde}}$ ,  $\mathbf{S}_{\text{tilde}}$ , and  $\mathbf{V}_{\text{tilde}}$ .
  - (d) Adjust the pattern of the signs of the columns of  $\mathbf{U}_{\text{tilde}}$  and  $\mathbf{V}_{\text{tilde}}$  to correspond to the pattern of  $\mathbf{U}$  (or, in this case, equivalently, to the pattern of  $\mathbf{V}$ ). Let  $\mathbf{A}_{\text{tilde}} = \mathbf{U}_{\text{tilde}} \cdot \mathbf{S}_{\text{tilde}} \cdot \mathbf{V}_{\text{tilde}}'$ .
  - (e) Compute the norms of  $\mathbf{U} - \mathbf{U}_{\text{tilde}}$ ,  $\mathbf{S} - \mathbf{S}_{\text{tilde}}$ ,  $\mathbf{V} - \mathbf{V}_{\text{tilde}}$ , and  $\mathbf{A} - \mathbf{A}_{\text{tilde}}$  and report  $\text{cond}(\mathbf{A})$ . Can you tell which matrices are computed stably?
  - (f) Repeat parts (c)–(d) for  $\mathbf{A} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}'$ . Which matrices are computed stably?
5. [0 marks] Prepare a 10–12 minute interim presentation on your project. As per the Progress Report Guidelines, your presentation you should clearly address the following aspects of your project:
- (a) the problem specification, including motivation and context (literature survey)
  - (b) the approach for obtaining a solution of the problem,
  - (c) the approach for verifying (and validating, if applicable) the results,
  - (d) any preliminary progress, including partial results or pilot data,
  - (e) metrics for measuring the success of the project.

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