

University of Saskatchewan
Department of Mathematics and Statistics

Numerical Analysis I
(MATH 211)

Instructor: Dr. Raymond J. Spiteri

ASSIGNMENT 01

Due: 8:30 a.m. Tuesday, January 15, 2013

1. **[15 marks]** In MATLAB, define $z = [10 \ 40 \ 20 \ 80 \ 30 \ 70 \ 60 \ 90]$.
 - (a) What vectors are specified by $z(1:2:7)$, $z(7:-2:1)$, and $z([3 \ 1 \ 4 \ 8 \ 1])$. Explain your answers.
 - (b) What is z after each of the following commands? Explain your answers.

```
z(1:2:7)          = zeros(1,4)
z(7:-2:1)         = zeros(1,4)
z([3 1 4 8 1])    = zeros(1,5)
```

2. **[15 marks]** Find the solutions to the equation

$$1 - 2x + 2x^2 - \frac{4x^3}{3} = -1$$

with the MATLAB functions `roots` and `solve`.

Use the `vpa` command to verify that the results are the same.

3. **[25 marks]** A *barn* is a unit of area equal to 10^{-28} m^2 used by nuclear physicists to measure effective cross-sections of very small particles.¹

Explain how this unit could be derived from the size of the atomic nucleus.

A *parsec* is a unit of length equal to 3.261584 light-years used by astronomers to measure faraway objects such as galaxies.²

Draw a figure to explain the concept of parallax and illustrate a parsec.

Given that the speed of light is defined to be 299,792,458 m/s and a teaspoon is 5 ml, write a MATLAB function

```
function tsp = bMpc2tsp(bMpc)
```

to convert barn-megaparsecs (bMpc) to teaspoons (tsp).

Use `bMpc2tsp` to give the equivalent in tsp of 3 bMpc.

¹In their experiments, the nuclear physicists use high-energy particle beams try to hit the side of a barn.

²A parsec is the distance that produces a parallax of one second.

4. [20 marks] Define a function $f(x)$ in MATLAB using a *function handle*

```
f = @(x) tan(sin(x)) - sin(tan(x))
```

Plot the function on the interval $[-\pi, \pi]$ using `ezplot`.

Create a plot of $f(x)$ vs. x for $x \in [-\pi, \pi]$ with spacing $\pi/256$. Add the labels `y` and `x` to the y - and x -axes, respectively, and the title “A wiggly function”.

Derive the supremum (least upper bound) of $f(x)$?

Show how to use the calculations performed in producing your plot to provide confidence in your calculation of the supremum.

5. [25 marks] The *golden ratio* (φ) has intrigued many people (not just mathematicians!) for over two millennia. The ratio is purported to occur naturally in many places, e.g., proportions within animal and human bodies, plants, and shells. The earliest deliberate use of the golden ratio by humans is considered to be made by Phidias (490–430 BC) in his construction of the Parthenon statues.

Give an illustration of the golden ratio in terms of line segments.

The golden ratio can be expressed as an infinite continued fraction:

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

We notice that the (overall) denominator of this expression is in fact φ ; hence we can write

$$\varphi = 1 + \frac{1}{\varphi}.$$

Solve this equation to derive an analytical formula for φ .

Starting from an initial iterate of `x = 1`, write a MATLAB script that implements a `while` loop to compute φ to within a tolerance `tol` by the iteration `x = 1 + 1/x`.

How many iterations does your loop take to converge to within `10eps` of φ ?