University of Saskatchewan
Department of Mathematics and Statistics

Numerical Analysis I
(MATH 211)
Instructor: Dr. Raymond J. Spiteri

ASSIGNMENT 02
Due: 8:30 a.m. Tuesday, February 05, 2013

1. [**10 marks**] Using the numbers

$$a = 2.3371258 \times 10^{-4}, \quad b = 3.3678429 \times 10^{2}, \quad c = -3.3677811 \times 10^{2},$$

and 8-digit arithmetic, show that floating-point addition ($\oplus$) is not associative, i.e.,

$$a \oplus (b \oplus c) \neq (a \oplus b) \oplus c.$$

You may assume that "8-digit arithmetic" means the answer is rounded off to 8 digits *after each operation*. For example, $1.2345678 + 0.12345678 = 1.3580246$.

2. [**10 marks**] On a certain computer, the distance between 7 and the next larger floating-point number is 2^{-12}. What is machine epsilon on that computer? What is the distance between 70 and the next larger floating-point number on that computer? Assume of course that the computer represents numbers in base 2.

3. [**40 marks**] You are working with a team of engineers on the design of a plane truss. The joint forces $\mathbf{x}$ are related to the applied forces $\mathbf{f}$ by $\mathbf{Ax} = \mathbf{f}$, where

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -1 & 7 \end{bmatrix}.$$

   (a) Use MATLAB's \ command to solve $\mathbf{Ax} = \mathbf{f}$, where $\mathbf{f} = [2\ 8\ 10]^T$.

   (b) Find the $\mathbf{LU}$ decomposition of $\mathbf{A}$ by Gaussian elimination using partial pivoting *by hand*. Show all your work, including all the elimination matrices $\mathbf{L}_k$, the permutation matrices $\mathbf{P}_k$, and the permuted elimination matrices $\mathbf{L}'_k$.

   (c) Verify your answer to the previous question using MATLAB's `lu` command.

   (d) Verify that there is no need to pivot the matrix $\mathbf{PA}$ when doing Gaussian elimination with partial pivoting.

4. [**10 marks**] Literally evaluate the polynomial

$$p(x) = x^3 - 6x^2 + 3x - 0.149$$

at $x = 4.71$ using 3-digit arithmetic. What is the relative error in your answer? Now evaluate $p(x)$ using Horner's algorithm. What is the relative error now? Is the more efficient calculation more accurate as well?

5. **[30 marks]** Explain how to compute the determinant of a matrix $\mathbf{A}$ by using its $\mathbf{LU}$ decomposition.

   It takes about $2m^3/3$ floating-point operations to compute the $\mathbf{LU}$ decomposition of a square matrix of dimension m. Suppose its determinant could be computed with m more multiplications.

   How long (in milliseconds) would it take a computer that can perform 1 billion $(= 10^9)$ multiplications per second to compute the determinant of a measly 20×20 matrix using this method?

   The way most people first learn to compute the determinant of a matrix is by expansion in minors. It can be shown that the number of multiplications required to evaluate the determinant of an $m \times m$ matrix in this way is $m!(m-1)$.

   How long (in years) would it take a computer that can perform 1 billion $(= 10^9)$ multiplications per second to compute the determinant of that same measly 20×20 matrix using this method?