

University of Saskatchewan
Department of Mathematics and Statistics

Numerical Analysis I
(MATH 211)

Instructor: Dr. Raymond J. Spiteri

ASSIGNMENT 03

Due: 8:30 a.m. Tuesday, February 26, 2013

1. [10 marks] Write down the roots of the polynomial: $W(x) = (x-1)(x-2) \dots (x-20)$. Now perturb one of the coefficients by a very small amount to create a new polynomial

$$p(x) = W(x) - 2^{-23}x^{19}. \quad (1)$$

Find all the roots of this new polynomial using MATLAB.

Is the problem of finding the roots of $W(x)$ sensitive? Explain.

2. [10 marks] Let \mathbf{A} , \mathbf{B} , and \mathbf{C} be $m \times m$ matrices, with \mathbf{B} and \mathbf{C} nonsingular, and let \mathbf{b} be an m -vector. Explain how you would compute \mathbf{x} from the formula

$$\mathbf{x} = \mathbf{B}^{-1}(2\mathbf{A} + \mathbf{I})(\mathbf{C}^{-1} + \mathbf{A})\mathbf{b}$$

without actually computing any inverses.

3. [10 marks] Write down the Lagrange form of the interpolating polynomial to the following data. (There is no need to simplify your expression.)

x	0	0.5	2	4.5	7
y	5	3.2	-1	-1.6	0

4. [30 marks] We are asked to provide an interpolant for the Bessel function of the first kind of order zero, $J_0(x)$.

(a) Create a table of data points listed to 7 decimal places for the interpolation points $x_1 = 1.0$, $x_2 = 1.3$, $x_3 = 1.6$, $x_4 = 1.9$, $x_5 = 2.2$.

[Hint: See MATLAB's help on `BesselJ`.]

(b) Fit a second-degree polynomial through the points x_1, x_2, x_3 . Use this interpolant to estimate $J_0(1.5)$. Compute the error.

(c) Fit a second-degree polynomial through the points x_3, x_4, x_5 . Using this to estimate $J_0(1.5)$ is called polynomial *extrapolation*. Explain the use of this term. Compute the error in using this polynomial to approximate $J_0(1.5)$.

(d) Fit a third-degree polynomial through the points x_2, x_3, x_4, x_5 . Use this interpolant to estimate $J_0(1.5)$. Compute the error.

(e) Fit a fourth-degree polynomial through the points x_1, x_2, x_3, x_4, x_5 . Use this interpolant to estimate $J_0(1.5)$. Compute the error.

(f) Does the highest-order interpolant give the best approximation to $J_0(1.5)$? Is this result surprising?

5. [40 marks] Consider the following population data for the United States:

Year	Population
1900	76 212 168
1910	92 228 496
1920	106 021 537
1930	123 202 624
1940	132 164 569
1950	151 325 798
1960	179 323 175
1970	203 302 031
1980	226 524 199

There is a unique polynomial of degree 8 that interpolates these 9 data points, but of course that polynomial can be represented in many different ways. Consider the following possible sets of basis functions $\phi_j(x)$, for $j = 1, 2, \dots, 9$:

- (i) $\phi_j(x) = x^{j-1}$ (the monomial basis)
- (ii) $\phi_j(x) = (x - 1900)^{j-1}$
- (iii) $\phi_j(x) = (x - 1940)^{j-1}$
- (iv) $\phi_j(x) = \left(\frac{x - 1940}{40}\right)^{j-1}$

- (a) For each of these four sets of basis functions, generate the corresponding Vandermonde matrix and compute an estimate of its condition number. How do the condition numbers compare? Explain your results by looking at the size of the entries in the Vandermonde matrices.
- (b) Use the most well-conditioned basis to compute the interpolating polynomial for the population data. Plot the polynomial and the data points.
- (c) Construct a cubic spline to interpolate the population data. Plot the spline and the data points.
- (d) Construct a `pchip` spline to interpolate the population data. Plot the spline and the data points.
- (e) Use the polynomial interpolant and both splines to extrapolate the population to 1990, and compare these values to the true value of 248 709 873. Which interpolant is the most accurate?