Mock Final Examination

Term: Fall 2005 (Sep 07 – Dec 05)

Student ID Information

Last name: ______________________ **First name:** ______________________

Student ID #: ____________________

Course ID:	**MATH 314**
Course Title:	Numerical Analysis III
Instructor:	Raymond Spiteri
Date of Exam:	25 December 2005
Time Period: Start:	14:00
End:	17:00
Duration of Exam:	3 hours
Number of Exam Pages: (including this cover sheet)	11 pages
Exam Type:	**Closed Book**
Additional Materials Allowed:	**Calculator**

Grade Table

Question	Score
1	/10
2	/20
3	/20
4	/30
5	/20
Total	/100

1. (10 points) Basic Theory

(a) [5 marks] Define what is meant by an initial-value problem to be *well-posed*. Does it make sense to try to compute the solution to a problem that is **not** well-posed? Explain.

(b) [5 marks] What are the two fundamental reasons why step-size selection and error control are critical parts of software for solving initial-value problems. Give examples to illustrate your reasons.

2. (20 points) One-step methods for IVPs

(a) [10 marks] Write down the backward Euler method for advancing a numerical solution to the test equation from $\mathbf{y}_n \approx \mathbf{y}(t_n)$ to $\mathbf{y}_{n+1} \approx \mathbf{y}(t_n + \Delta t)$.

Explain why functional iteration is a bad way to solve these equations if an initial value problem is stiff.

Show how you would solve these equations for a stiff problem.

(b) [10 marks]

(i) Explain the reasoning behind the naming of the initial value problem solver `ode45`. How does this differ from the reasoning behind the naming of the initial value problem solver `ode113`?

(ii) Explain what is meant when a Runge–Kutta method is referred to as having the *first-same-as-last* (FSAL) property? Why is it advantageous for a Runge–Kutta method to have the FSAL property?

3. (20 points) Multi-step methods for IVPs

(a) [10 marks]

(i) Name two advantages that linear multi-step methods have compared to Runge–Kutta methods.

(ii) Name two disadvantages that linear multi-step methods have compared to Runge–Kutta methods. Explain how these disadvantages are handled by the popular software packages.

(b) [10 marks] Carefully explain how *local extrapolation* is used in the context of a standard implementation of a predictor-corrector pair. Carefully explain how local extrapolation is **not** used in the context of a predictor-corrector pair that uses Milne's device.

4. (30 points) Boundary-value problems

(a) [10 marks] A boundary value problem is solved 4 times with `bvp4c` using the following sets of options:

1. `Vectorized` set to `'on'`.
2. Analytical partial derivatives are used.
3. `Vectorized` set to `'on'` and analytical partial derivatives are used.
4. The default settings are used.

Put these 4 scenarios in (typical) order of slowest to fastest in terms of time required to solve the problem.

Name two important advantages of supplying analytical partial derivatives to `bvp4c`.

(b) [10 marks] Carefully explain what is meant by *collocation*. Why is bvp4c classified as a collocation code?

(c) [10 marks] Convert the following multi-point boundary value problem to a form suitable for input to `bvp4c`:

$$y''(x) + \lambda y(x) = 0,\ \ y(0) = 0, y(1) = 1, y(c) = y'(c),$$

where $y(x)$ is the unknown function and λ and c are unknown parameters.

Note: You may assume that the problem is well-posed and that $c < 1$.

5. (20 points) Special Topics

(a) [10 marks] The following initial value problem (IVP) describes space charge current in a cylindrical capacitor

$$y(y'')^2 = e^{2x}, \quad y(0) = 0,\ y'(0) = 1,\ t \in [0, 0.1].$$

Convert this IVP to a first-order form suitable for solution with a MATLAB IVP solver such as `ode15s`.

Where is this IVP singular?

Explain how you would combine an analytical (series) approximation to the solution near this singular point to obtain a numerical solution over the entire time interval.

(b) [10 marks] What is an *event function*? Explain how you would use an event function to solve the following initial value problem that models the behaviour of a thermostat

$$\dot{y} = \begin{cases} -\frac{y}{2} & \text{if } y \geq Y_E, \\ 10 - \frac{y}{2} & \text{if } y < Y_E, \end{cases}$$

where $y(t)$ is the temperature in the room, and Y_E is the desired temperature. You may assume that the initial condition is $y(0) = Y_E/2$ and that the integration is to be carried out on the interval $t \in [0, 20]$.