Angle-Monotone Graphs: Construction and Local Routing

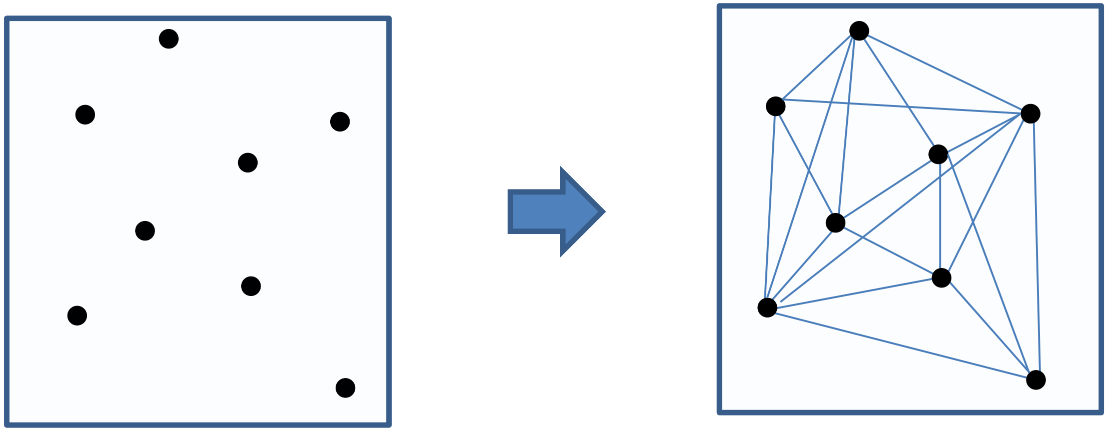
Anna Lubiw
Cheriton School of Computer Science, University of Waterloo, Canada

<u>Debajyoti Mondal</u>
Department of Computer Science, University of Saskatchewan, Canada

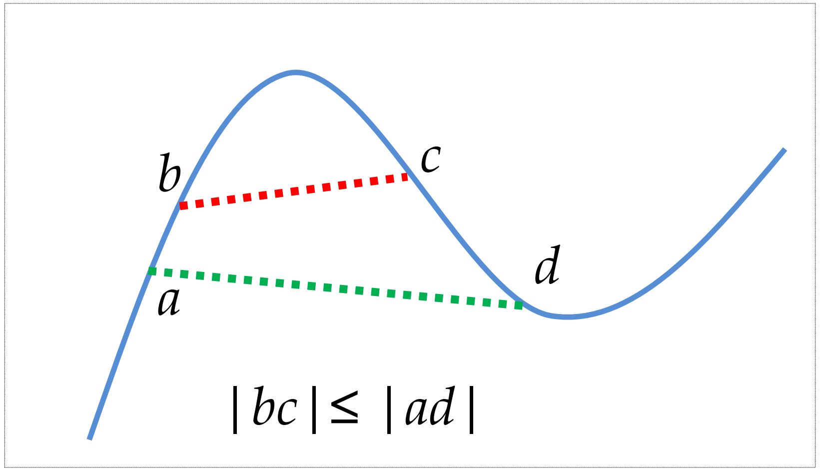
July 28, 2018

WG 2018

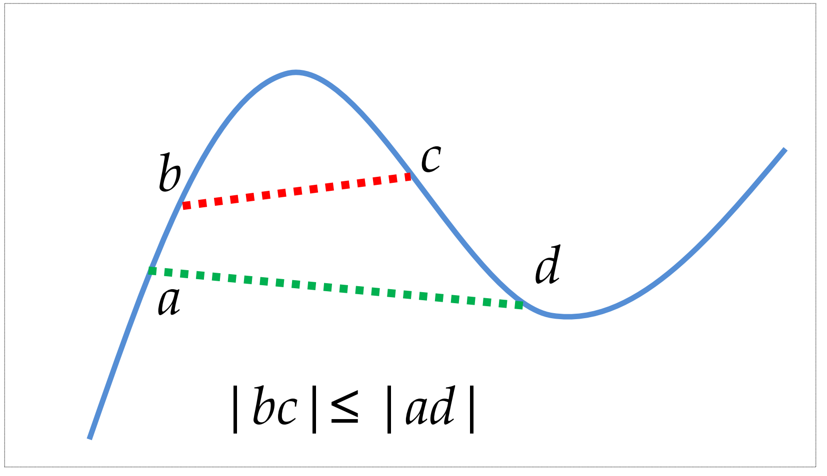
Given a point set, Construct a 'Good' Graph

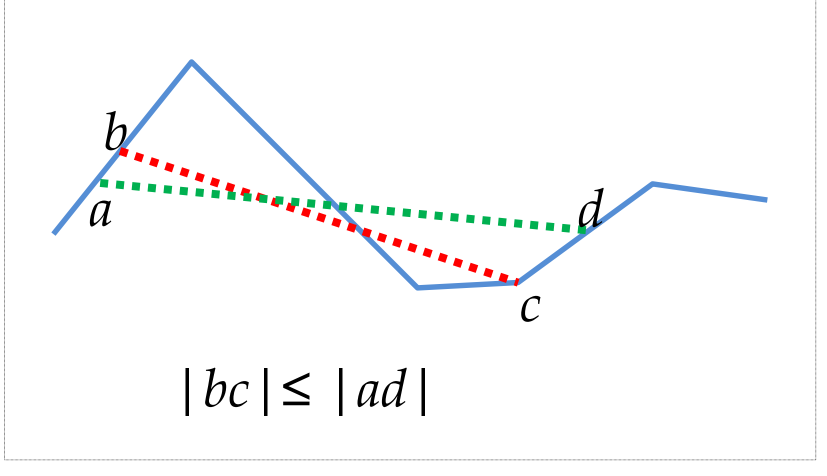


Increasing-Chord Paths

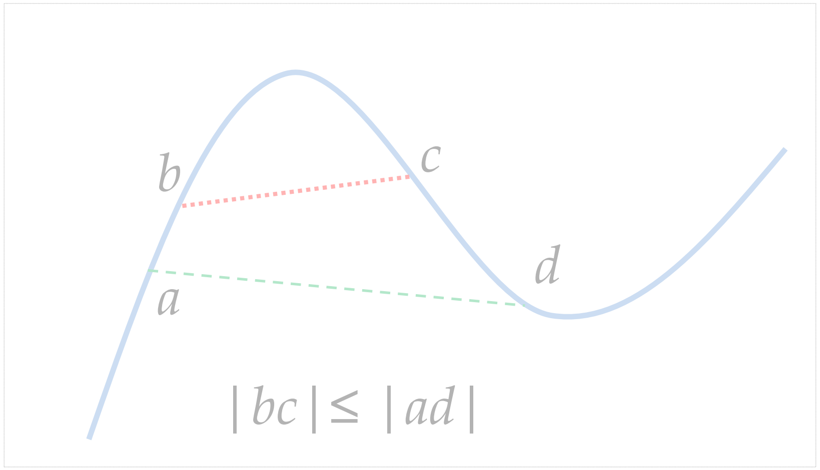


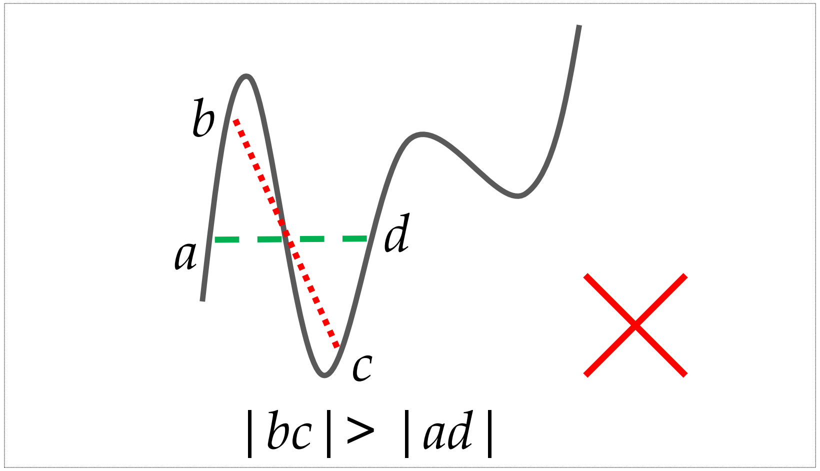
Increasing-Chord Paths

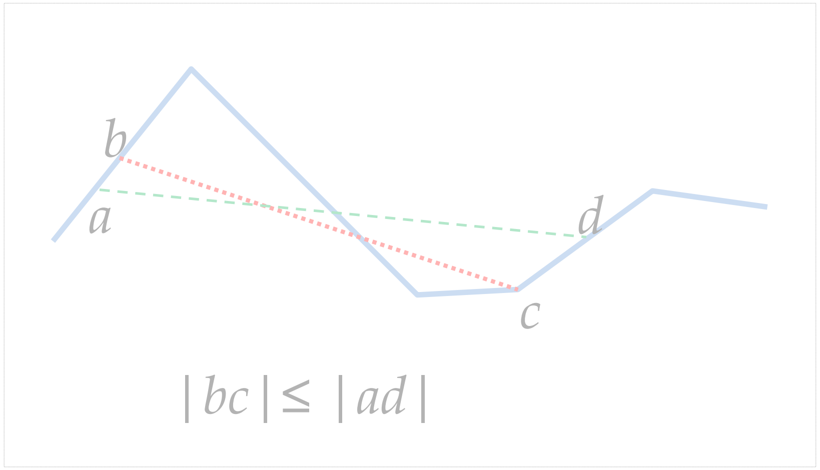


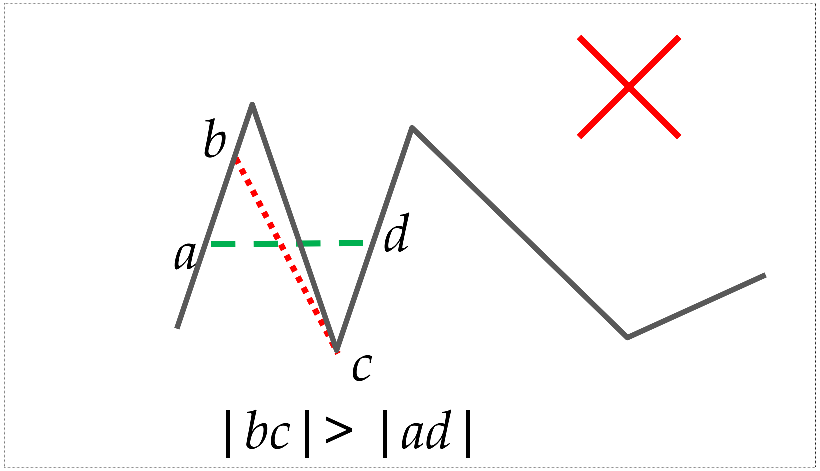


Increasing-Chord Paths

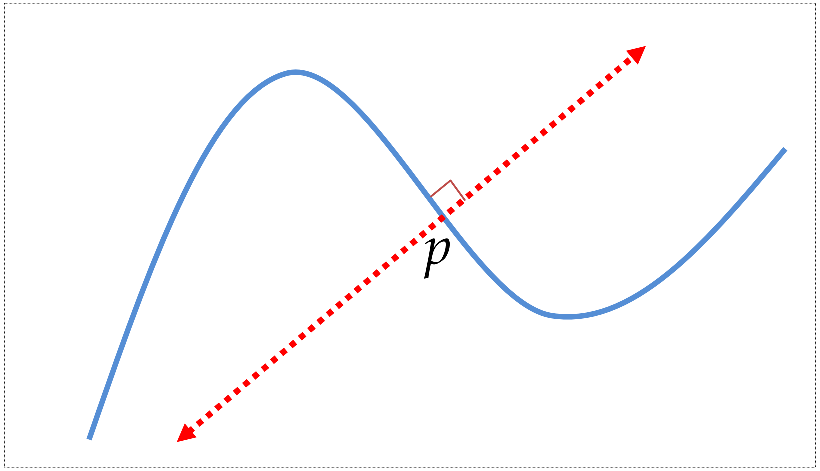


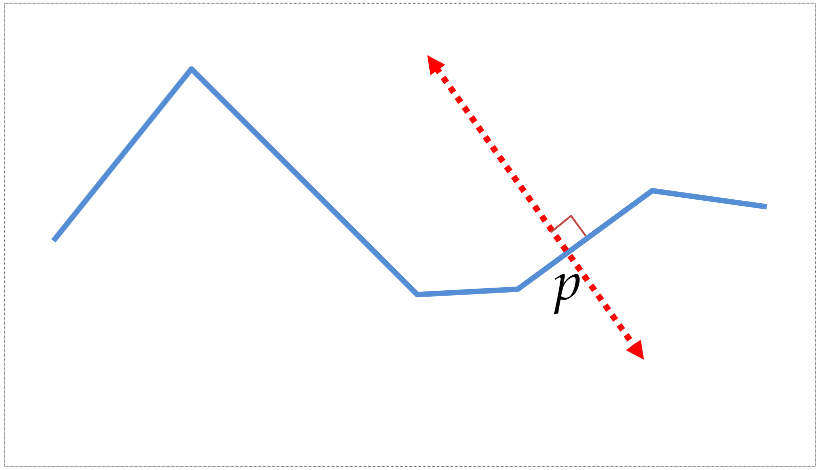






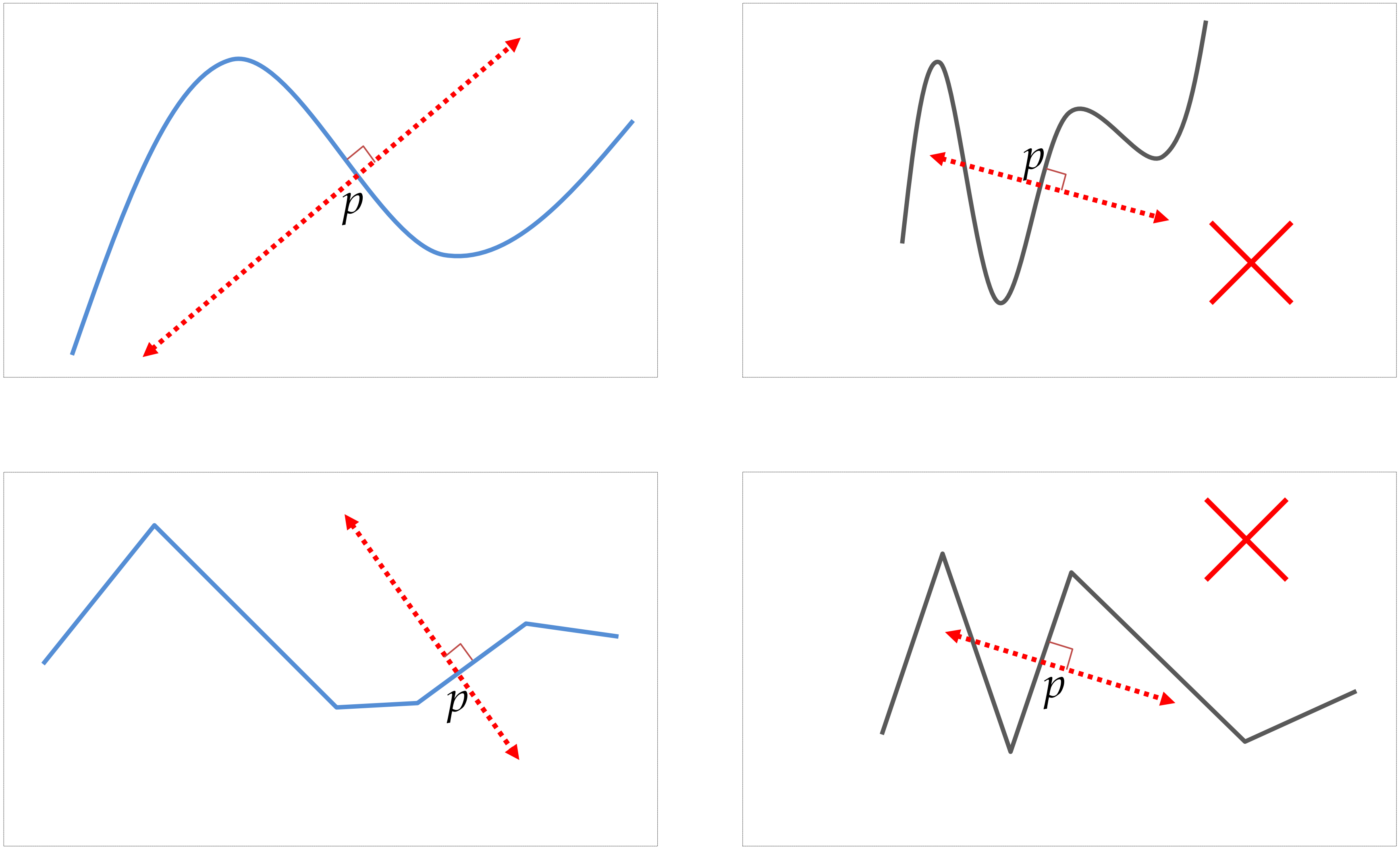
Increasing-Chord Paths





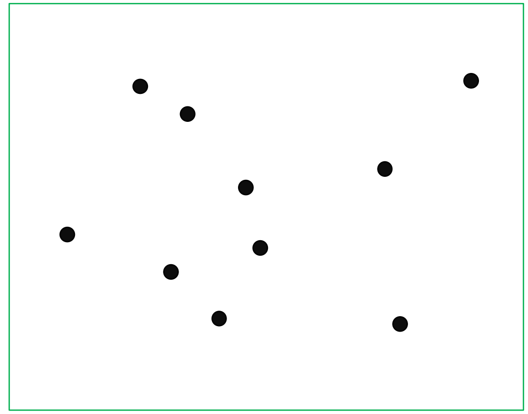
[Icking et al.'95]

Increasing-Chord Paths



[Icking et al.'95]

Related Research



Construct a small spanning geometric graph that is

- *Monotone* [Mastakas and Symvonis, '17]
- *Good spanner - planar* [Chew '89; Dobkin et al, '90; Bose et al. '10, Xia, '13]
- *Good spanner - nonplanar* [Narasimhan and Smid (Book) 2007]
- Increasing-chord [Alamdari et al.'13; Dehkordi et al.'15; Bonichon et al.'16]

Open Question

[Dehkordi, Frati, and Gudmundsson, 2015]

Input: A point set P

Question: Does there exist an increasing-chord graph on P with $o(n^2)$ edges?

Open Question

[Dehkordi, Frati, and Gudmundsson, 2015]

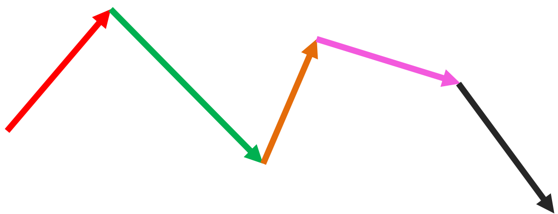
Input: A point set P

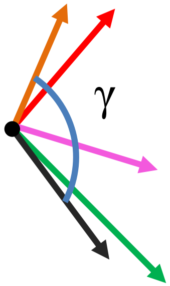
Question: Does there exist an increasing-chord graph on P with $o(n^2)$ edges?

This presentation

It suffices to construct an **angle-monotone graph of width 90°** with subquadratic number of edges.

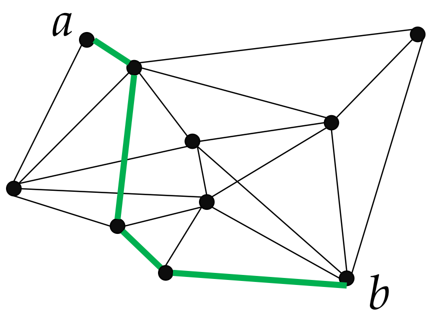
Angle Monotone Path of width γ





A geometric graph in the plane is angle-monotone of width γ if every pair of vertices is connected by an angle-monotone path of width γ.

Graphs & Dilation



$$\frac{\text{shortest path distance between } a \text{ and } b}{|ab|} \leq (1/\cos(\gamma/2))$$

If $\gamma = 90^o$, then the graph is a $\sqrt{2}$ spanner

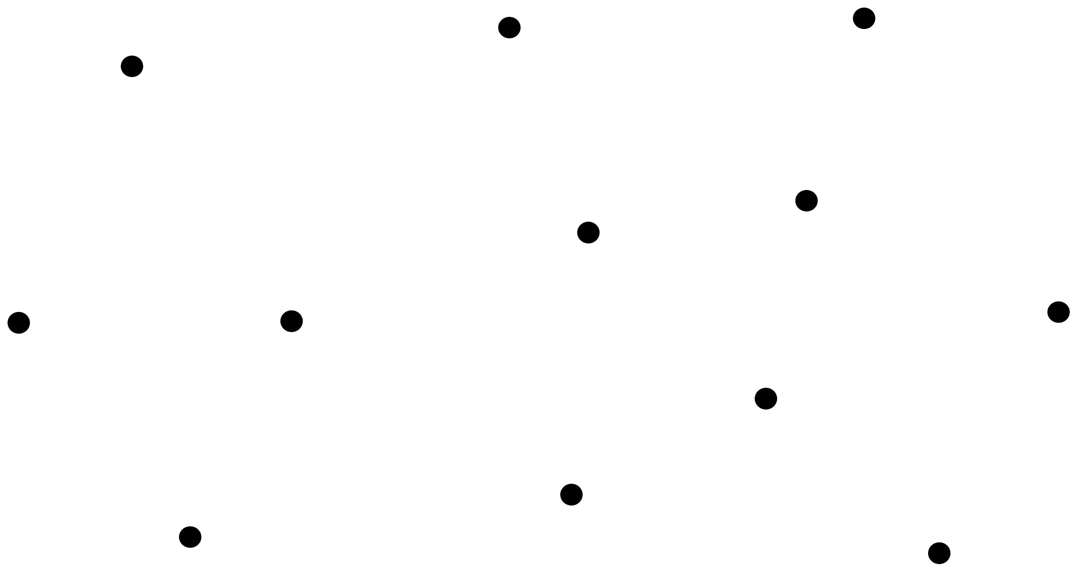
Angle-monotone graphs of width 90^o are increasing chord graphs.

This presentation

- We can construct an angle-monotone graph of width 90° with $O(n^2 \log\log n / \log n)$ edges
- We can construct an angle-monotone graph of width $(90^\circ+\alpha)$ with $O(n/\alpha)$ edges.
- We also have a 2-local routing algorithm that can find a path of width $(90^\circ+\alpha)$

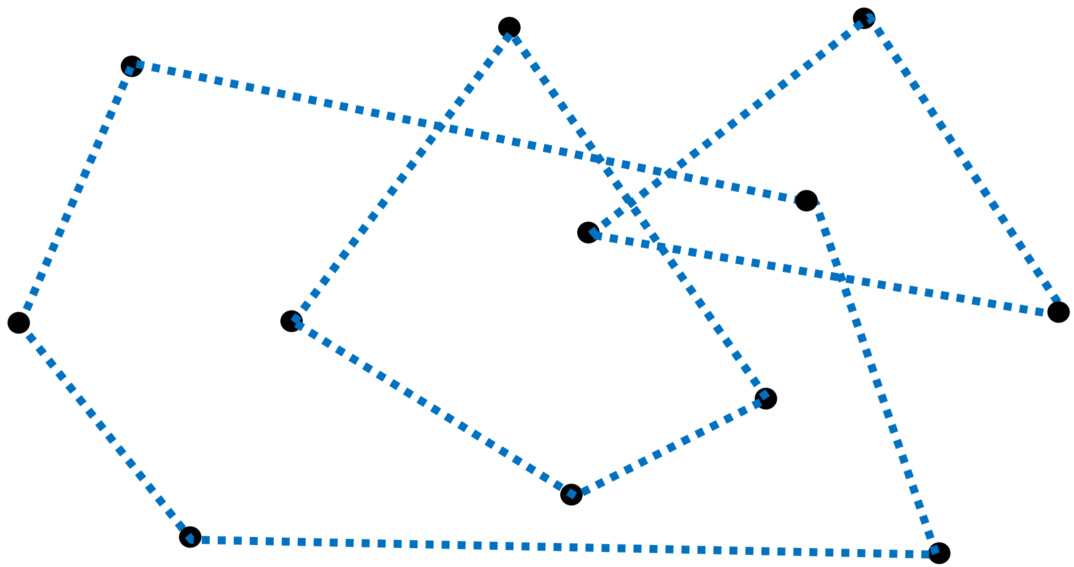
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log \log n / \log n)$ edges

- Partition the point set into $O(n/\log n)$ convex polygons, each of size $O(\log n)$ [Urabe 1996]



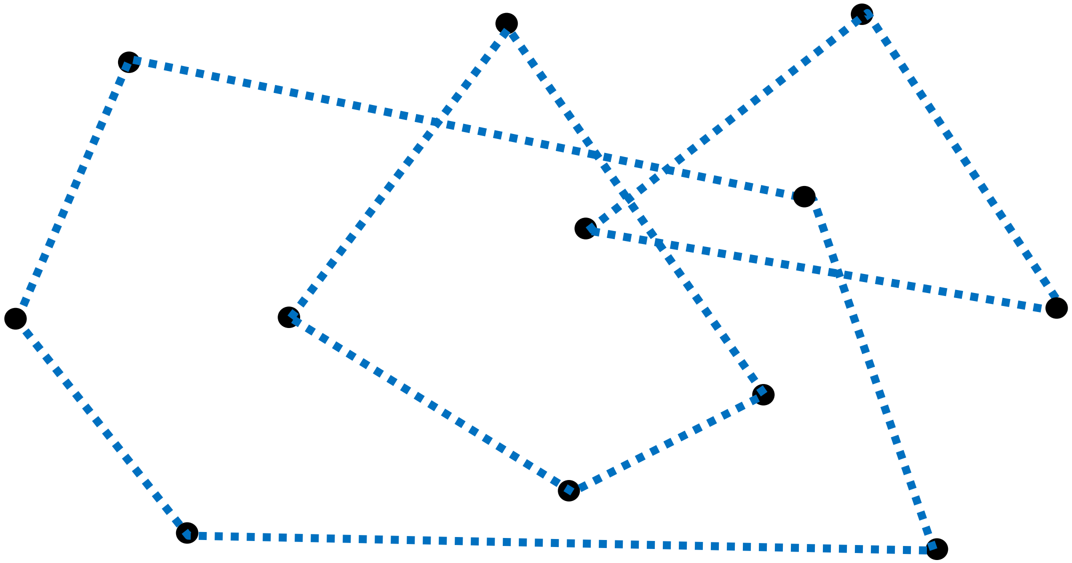
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- Partition the point set into $O(n/\log n)$ convex polygons, each of size $O(\log n)$ [Urabe 1996]



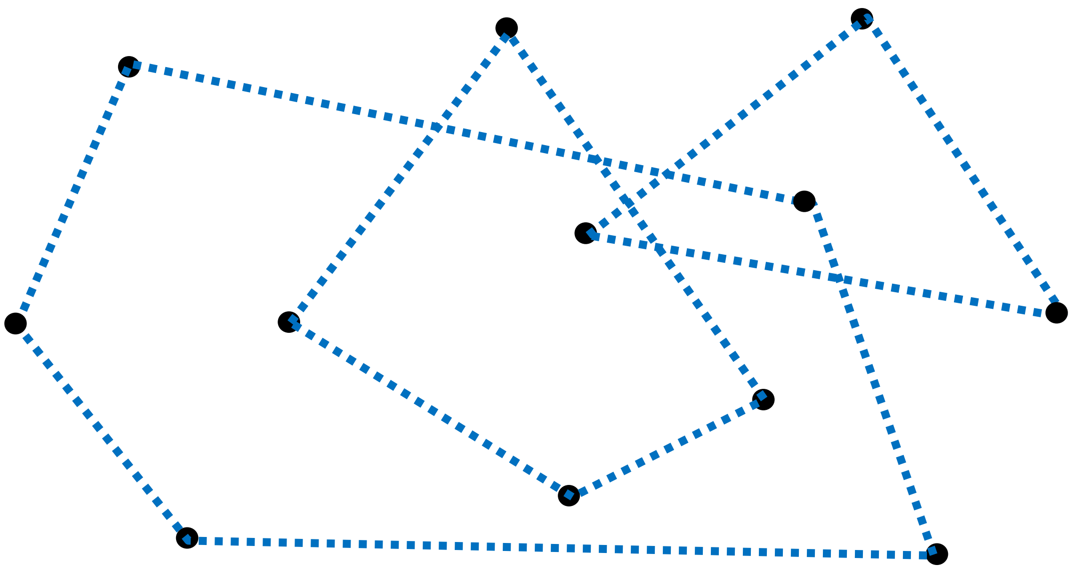
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- Partition the point set into $O(n/\log n)$ convex polygons, each of size $O(\log n)$ [Urabe 1996]
- Add all possible edges within and between these convex polygons: $O((n/\log n + n^2/\log^2 n)\log^2 n)$



Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log \log n / \log n)$ edges

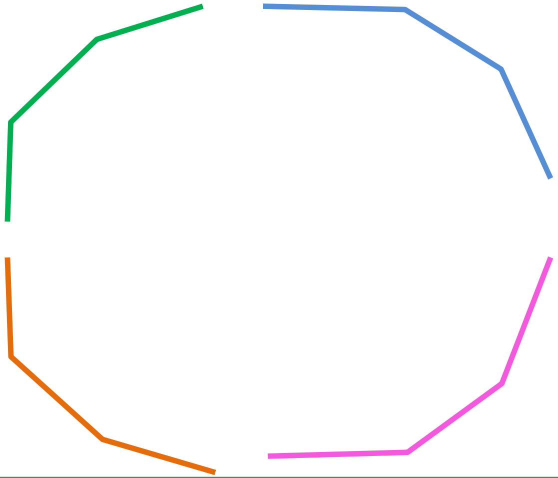
- Partition the point set into $O(n/\log n)$ convex polygons, each of size $O(\log n)$ [Urabe 1996]
- Add all possible edges within and between these convex polygons: $O((n/\log n + n^2 / \log^2 n) \log^2 n)$



Does not work.

Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- Partition the point set into $O(n/\log n)$ convex polygons, each of size $O(\log n)$ [Urabe 1996]
- For each convex polygon, construct an angle monotone graph: $O(n \log n)$ edges in total.
- Partition each of these convex polygons into
  a (x, y)-convex path,
  a (x, -y)-convex path,
  a (x, y)-concave path, and
  a (x, -y)-concave path.



Construction of Angle-Monotone Graphs
width 90°, O(n^2 log log n / log n) edges

- For every pair of paths, we now construct an angle-monotone graph spanning these paths.

Construction of Angle-Monotone Graphs
width 90°, O(n^2 log log n / log n) edges

- For every pair of paths, we now construct an angle-monotone graph spanning these paths.

Case Analysis :

a (x, y)-convex path
a (x, y)-convex path

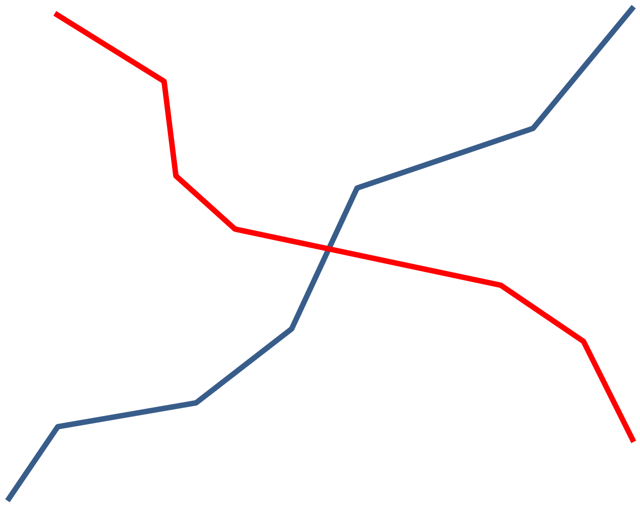
a (x, -y)-convex path
a (x, y)-convex path

a (x, -y)-convex path
a (x, y)-concave path

⋮

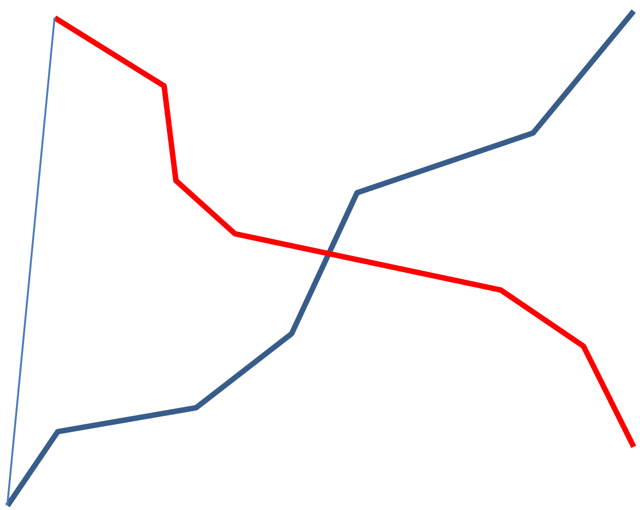
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



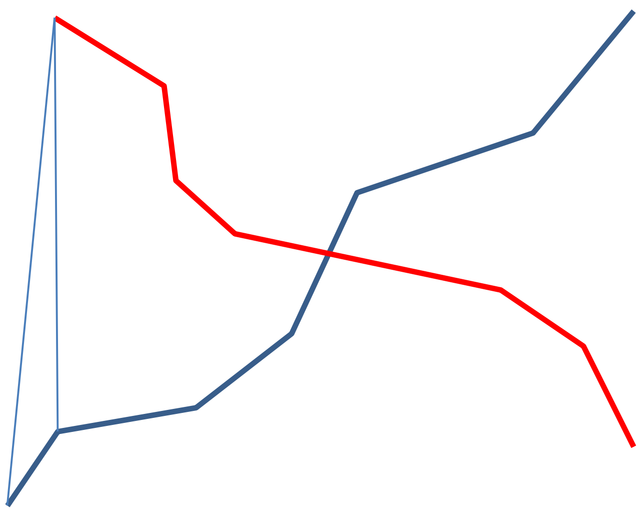
Construction of Angle-Monotone Graphs
width 90°, O(n^2 log log n / log n) edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



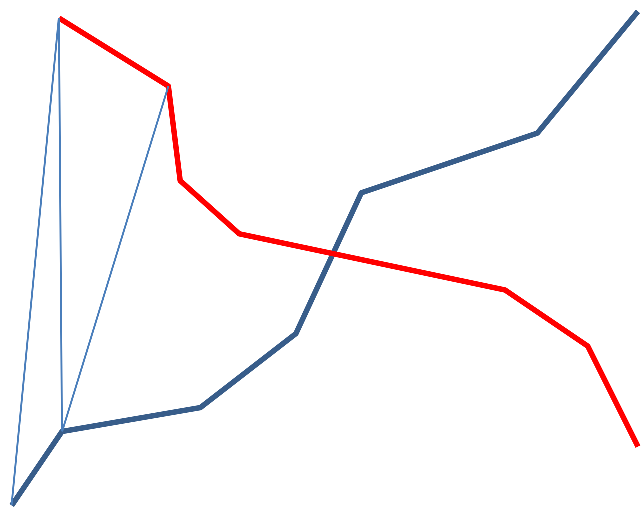
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



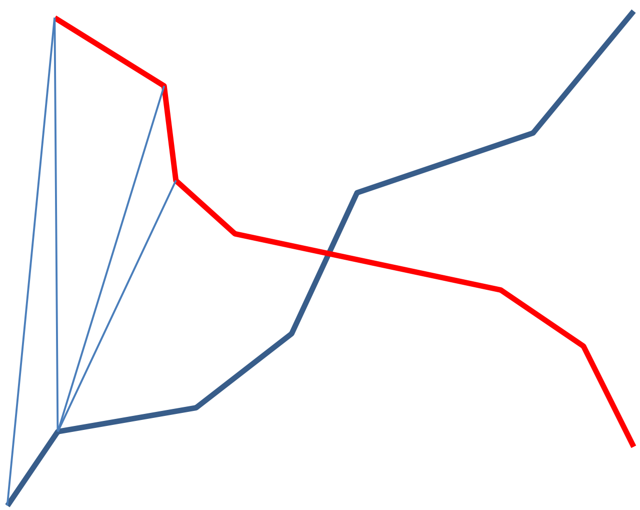
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



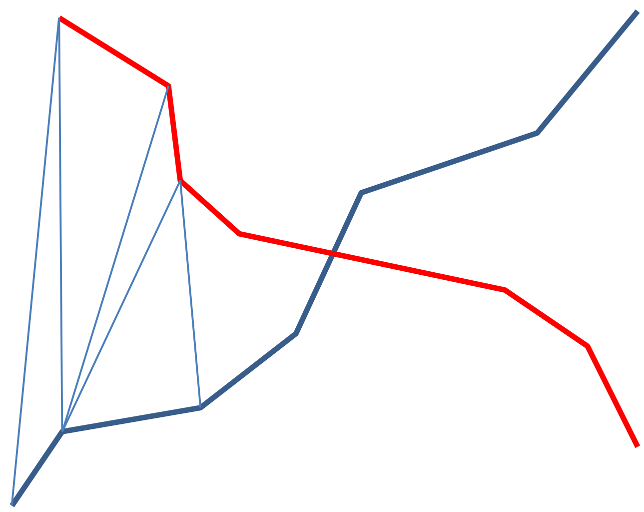
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



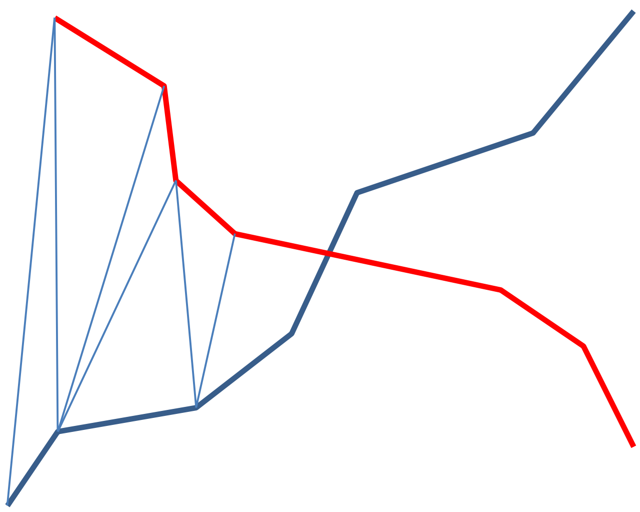
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log \log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



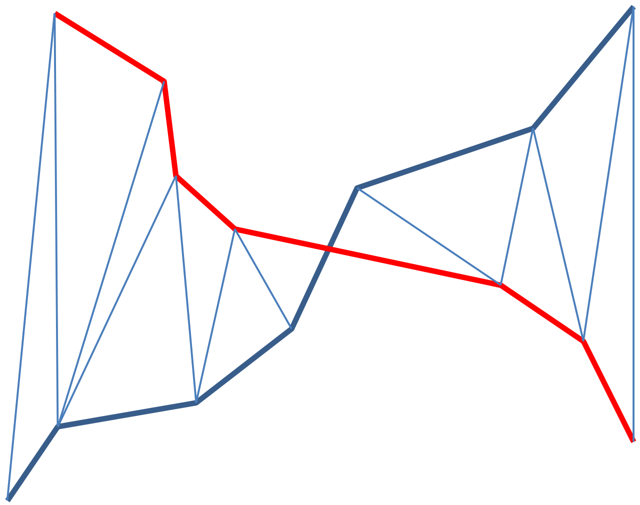
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



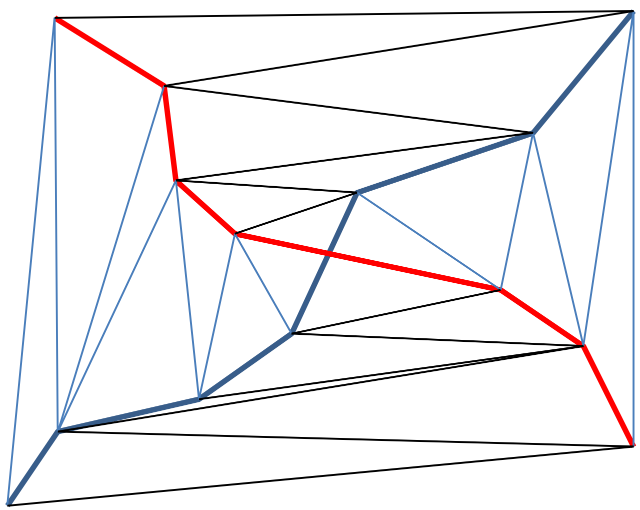
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



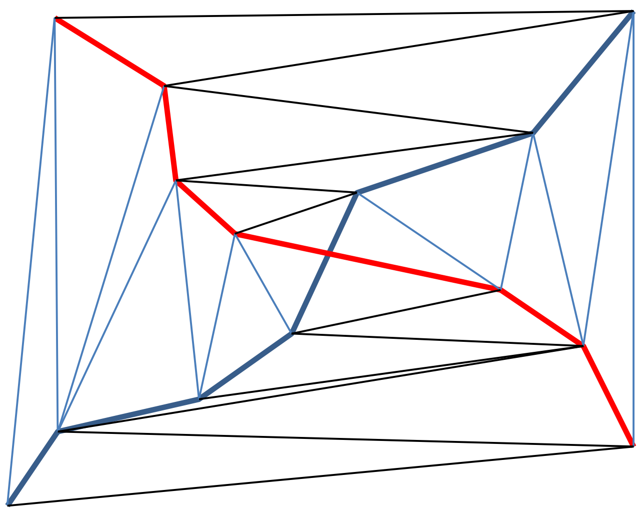
Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log \log n / \log n)$ edges

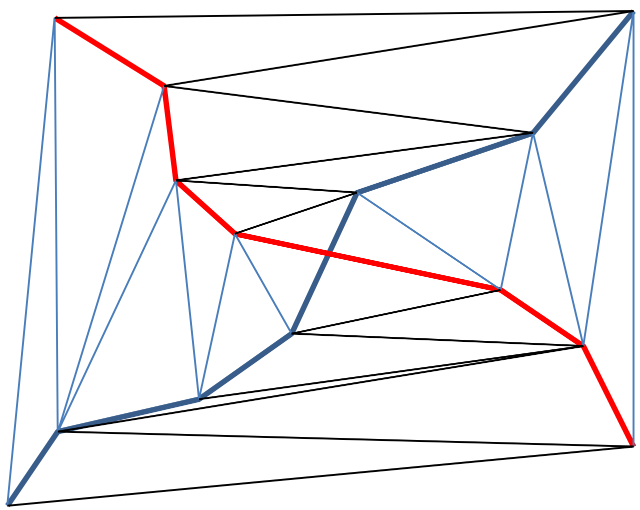
- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



An angle-monotone graph of linear size

Construction of Angle-Monotone Graphs
width 90°, $O(n^2 \log\log n / \log n)$ edges

- An example in more general setting
  - Assume that you are given a (x,y)-monotone and a (x,-y)-monotone path



This already covers all combinations except for when both paths are of type (x,y), or both are of type (x,-y).

An angle-monotone graph of linear size

Construction of Angle-Monotone Graphs

width 90°, $O(n^2 \log\log n / \log n)$ edges

- For every pair of paths, we now construct an angle-monotone graph spanning these paths.
- In each case we add at most $O(\log n \log\log n)$ edges

$O(n^2/\log^2 n) \times O(\log n \log\log n)$

Case Analysis :

a (x, y)-convex path
a (x, y)-convex path

a (x, -y)-convex path
a (x, y)-convex path

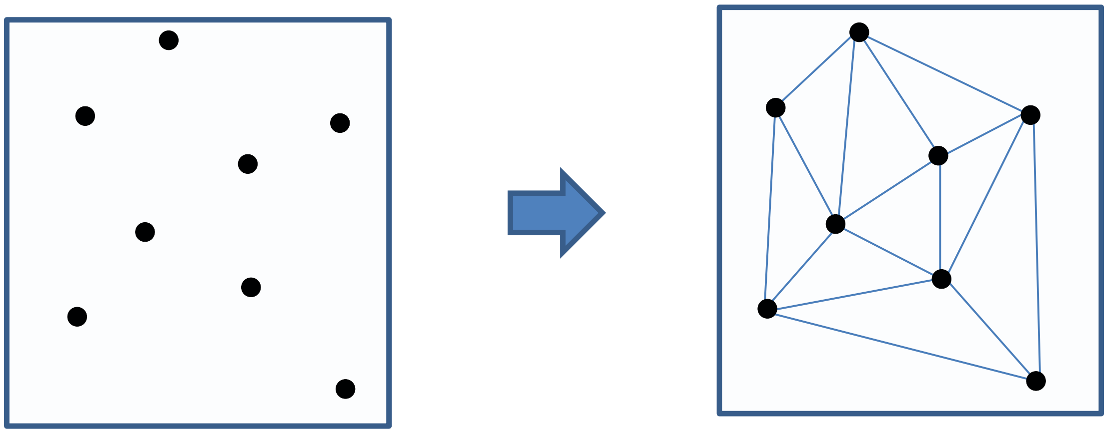
a (x, -y)-convex path
a (x, y)-concave path

⋮

Construction of Angle-Monotone Graphs width $(90^{\circ}+\alpha)$ with $O(n/\alpha)$ edges

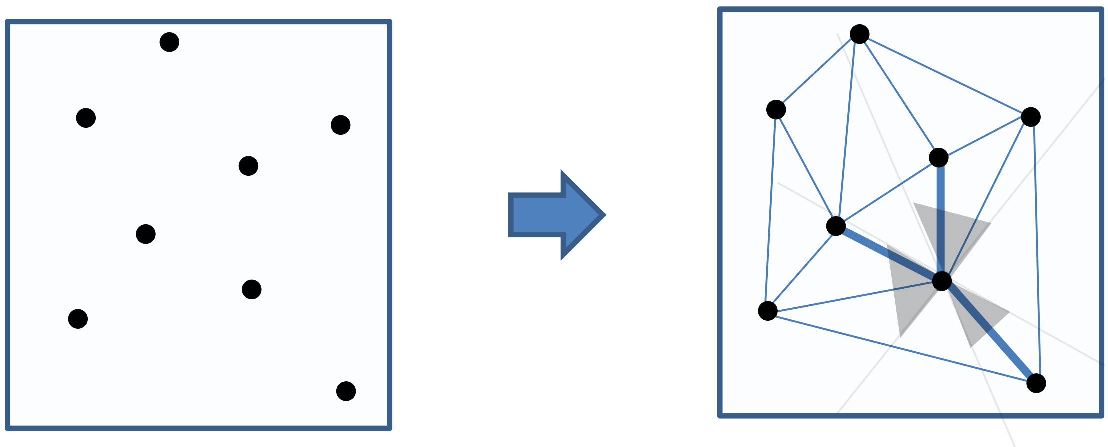
Construction of Angle-Monotone Graphs width (90°+α) with O(n/α) edges

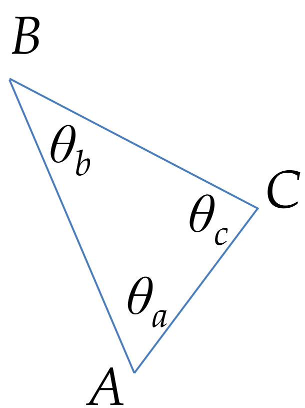
A 3-sweep graph on a point set

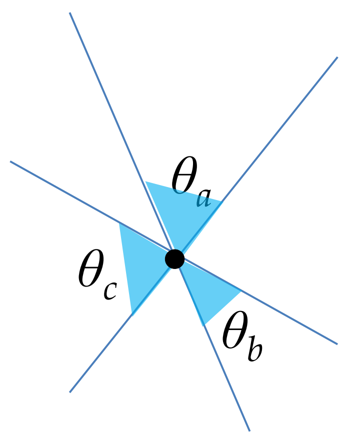


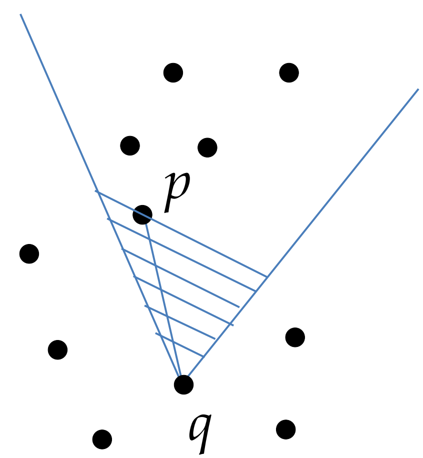
Construction of Angle-Monotone Graphs width $(90^\circ+\alpha)$ with $O(n/\alpha)$ edges

A 3-sweep graph on a point set



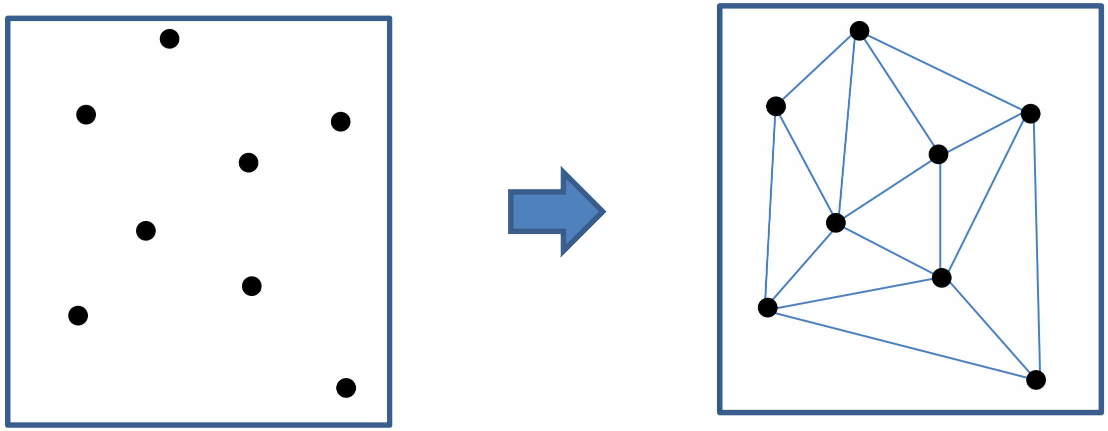






Construction of Angle-Monotone Graphs width (90°+α) with O(*n*/α) edges

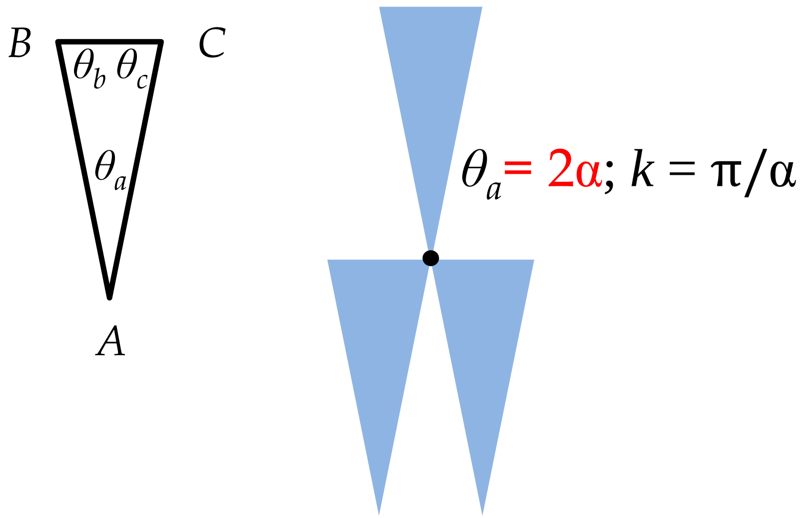
A 3-sweep graph on a point set

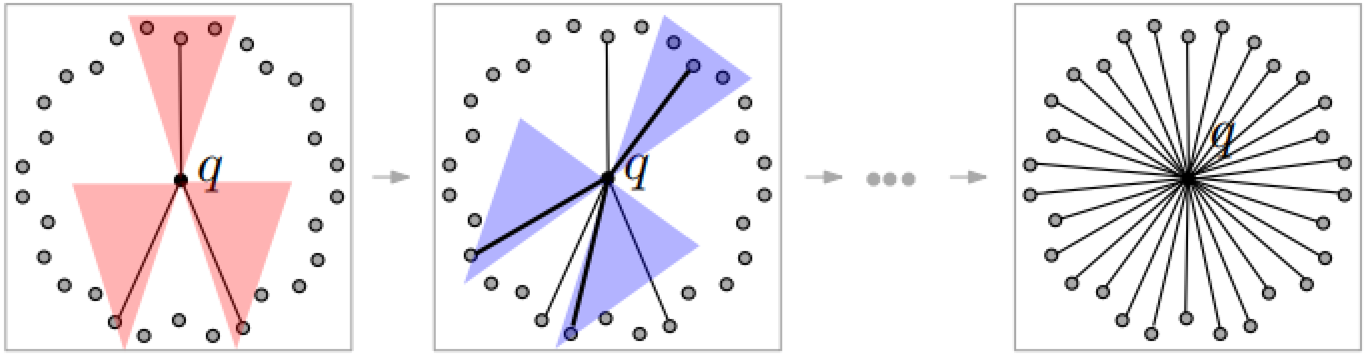


- Planar
- If $\theta_a = \theta_b = \theta_c = 60^o$, then the graph coincides with the well-known half-theta-6-graph.

Construction of Angle-Monotone Graphs width (90°+α) with O(n/α) edges

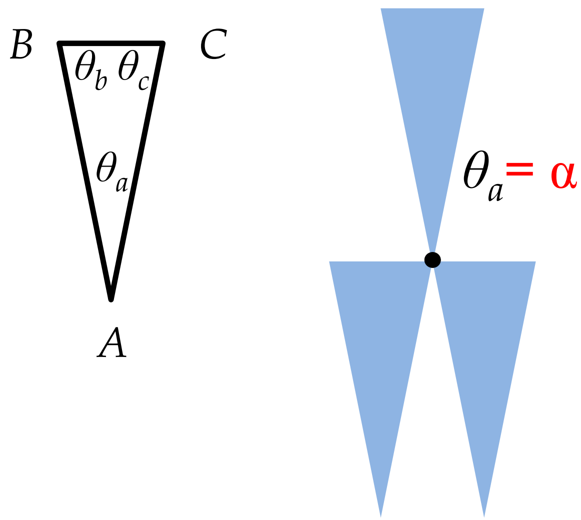
A *k*-layer 3-sweep graph on a point set



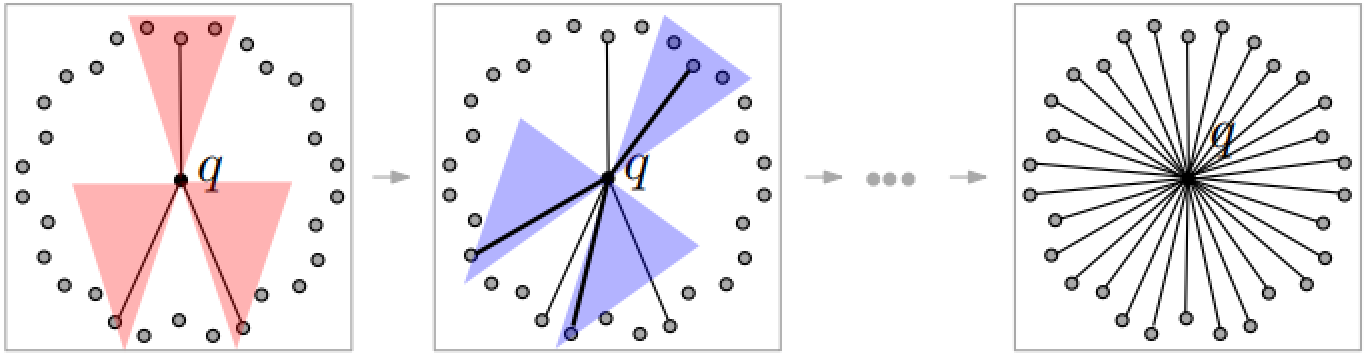


Construction of Angle-Monotone Graphs width $(90^{\circ}+\alpha)$ with $O(n/\alpha)$ edges

A *k*-layer 3-sweep graph on a point set



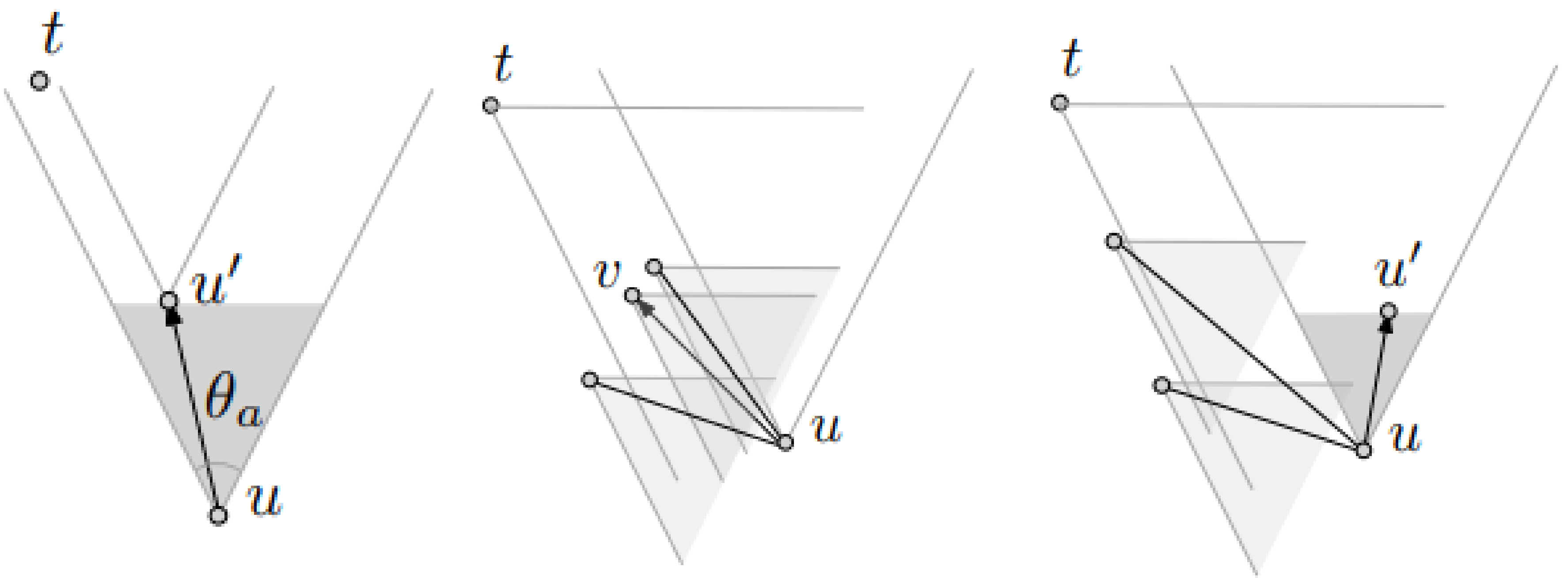
May have high degree but still a union of small number of planar graphs



2-Local Routing on *k*-layer 3-sweep graphs

- Send a message from the source to the destination
- The current vertex knows
  - The locations of its neighbors and their neighbors (2-hop distance).
  - The location of the destination.
  - Does not use additional memory (e.g., no routing table)

2-Local Routing on *k*-layer 3-sweep graphs



- Routing Rules
- The path taken is an angle monotone path of width $(90^\circ+\alpha)$

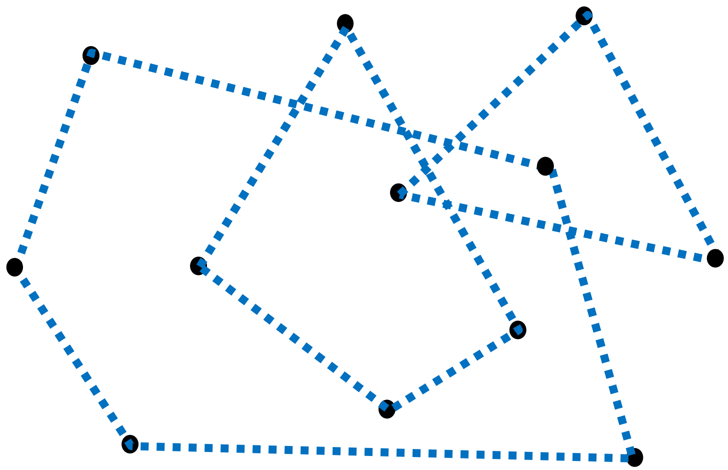
Open Questions

Does every point set admit an angle-monotone graph with linear number of edges?

Find the smallest γ such that every point set has a planar angle-monotone graph of width γ.

Currently it is known that $90^\circ < \gamma \leq 120^\circ$.

Angle-Monotone Graphs: Construction and Local Routing



Thank You