Drawing Planar Graphs with Reduced Height

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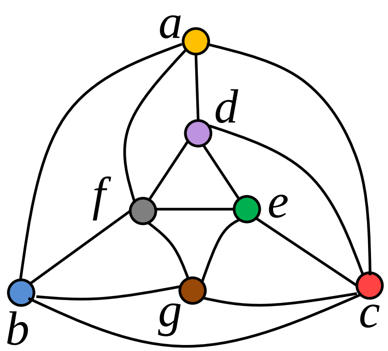
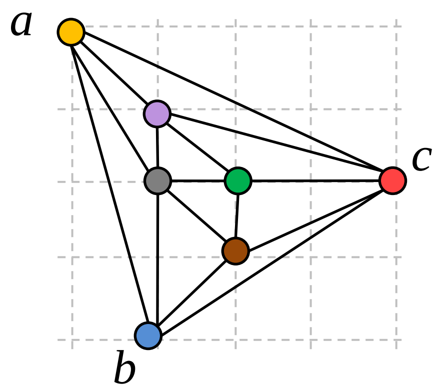
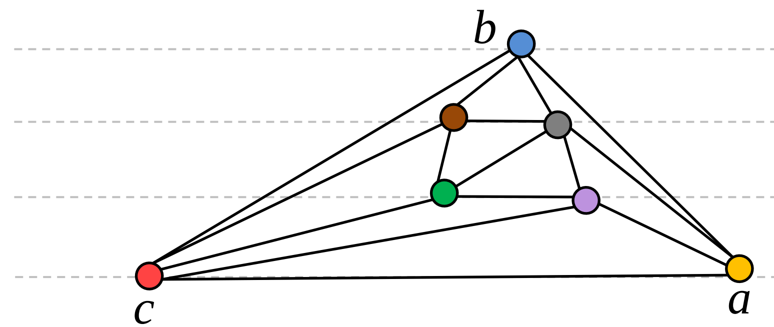
Straight-line Drawings (Fixed Vs. Variable)



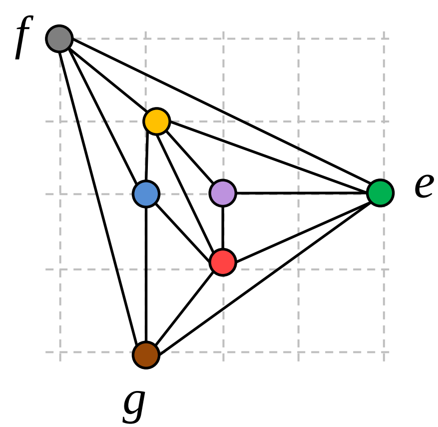
A planar graph G

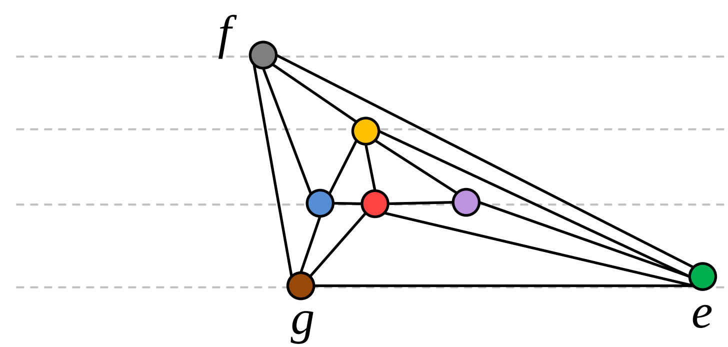


A straight-line drawing of *G* on a 5×5 grid

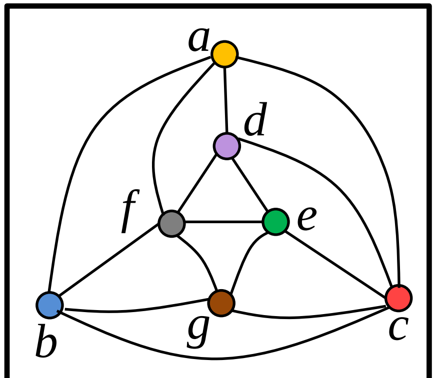


A straight-line drawing of *G* with height 4

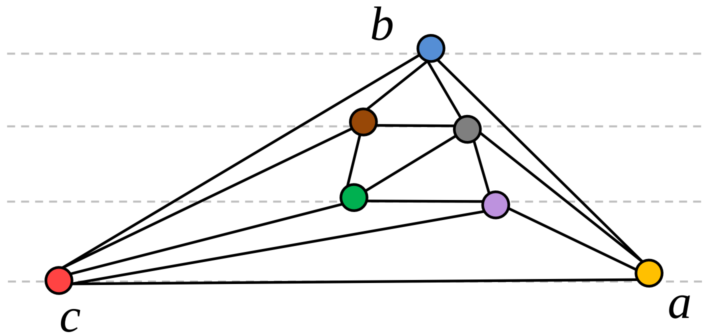


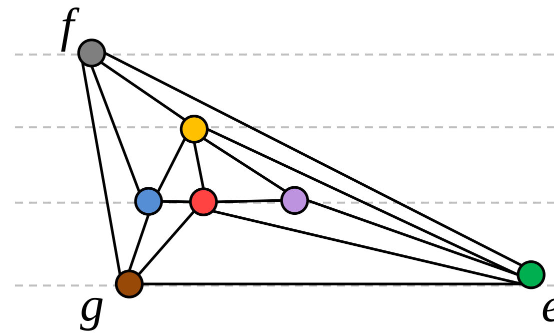


Straight-line Drawings (Fixed Vs. Variable)

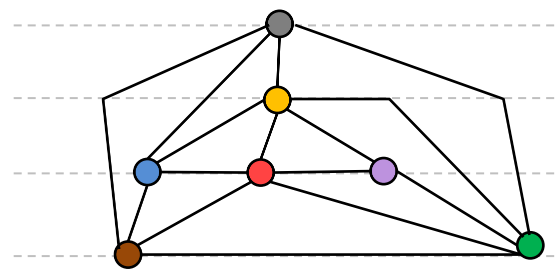


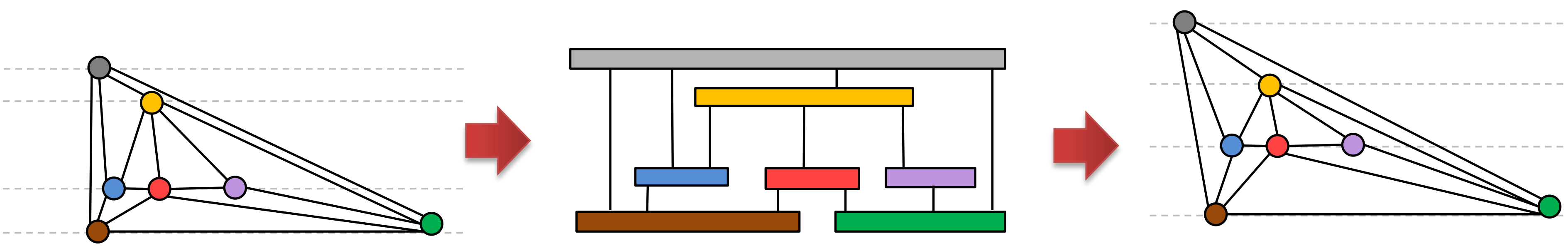
A planar graph G





Here we allow variable embedding.
Sometimes we use edge-bends.
Focus on height.





Via visibility representation
Biedl, 2014

Fixed Embedding

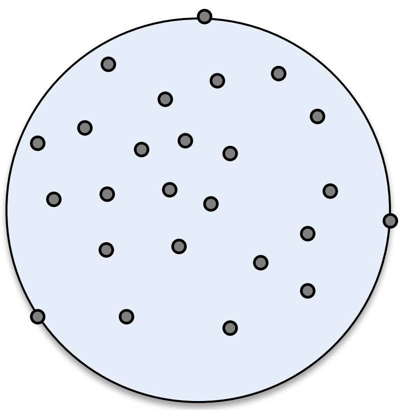
Upper Bounds	Lower Bounds
Triangulations Area / Height $2n^2 + O(1)$ / $n - 2$ [de Fraysseix et al. 1990] $n^2 + O(1)$ / $n - 2$ [Schnyder 1990] $1.78n^2 + O(1)$ / $0.66n$ [Chrobak and Nakano 1998] $0.88n^2 + O(1)$ / $0.66n$ [Brandenburg 2008] $0.44n^2 + O(1)$ / $0.66n$ (polyline) [Bonichon et al. 2003]	Nested Triangles Graph Area / Height $0.44n^2 + O(1)$ / $0.66n$ [Dolev et al. 1984] A class of planar 3-trees Area / Height $0.44n^2 + O(1)$ / $0.66n$ [Frati and Patrignani 2008, Mondal et al. 2010]

Variable Embedding

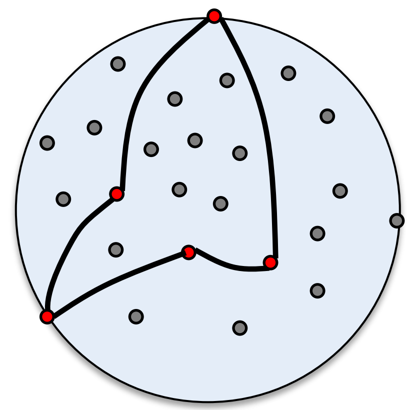
Upper Bounds	Improved Upper Bounds
Triangulations Area / Height $0.88n^2 + O(1)$ / $0.66n$ [Brandenburg 2008] $0.44n^2 + O(1)$ / $0.66n$ (polyline) [Bonichon et al. 2003] Planar 3-trees Area / Height $0.88n^2 + O(1)$ / $0.5n$ [Brandenburg 2008, Hossain et al. 2013] Nested Triangles Graph Area / Height $0.22n^2 + O(1)$ / $0.33n$ [Frati and Patrignani 2008]	Triangulations Polyline drawing with height $4n/9+O(\lambda\Delta) \approx 0.44n+O(\lambda\Delta)$ This is $0.44n+o(n)$ when Δ is $o(n)$ Planar 3-trees Straight-line drawing with height $4n/9+O(1) \approx 0.44n+O(1)$

Idea: Use the Simple Cycle Separator

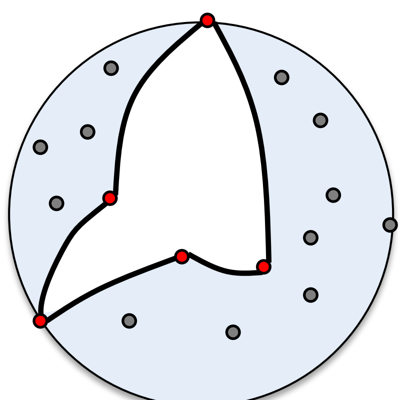
[Djidjev and Venkatesan, 1997]
Every planar triangulation has a simple cycle separator of size $O(\sqrt{n})$



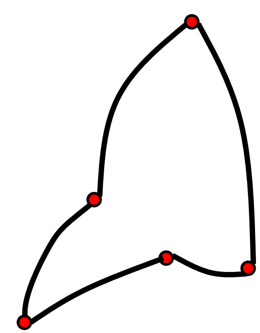
An n-vertex planar graph G



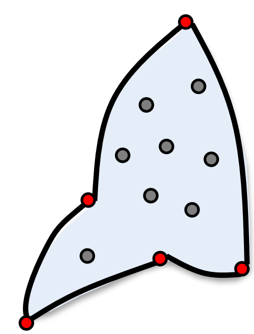
A simple cycle separator of G



G_o with $2n/3+O(\sqrt{n})$ vertices

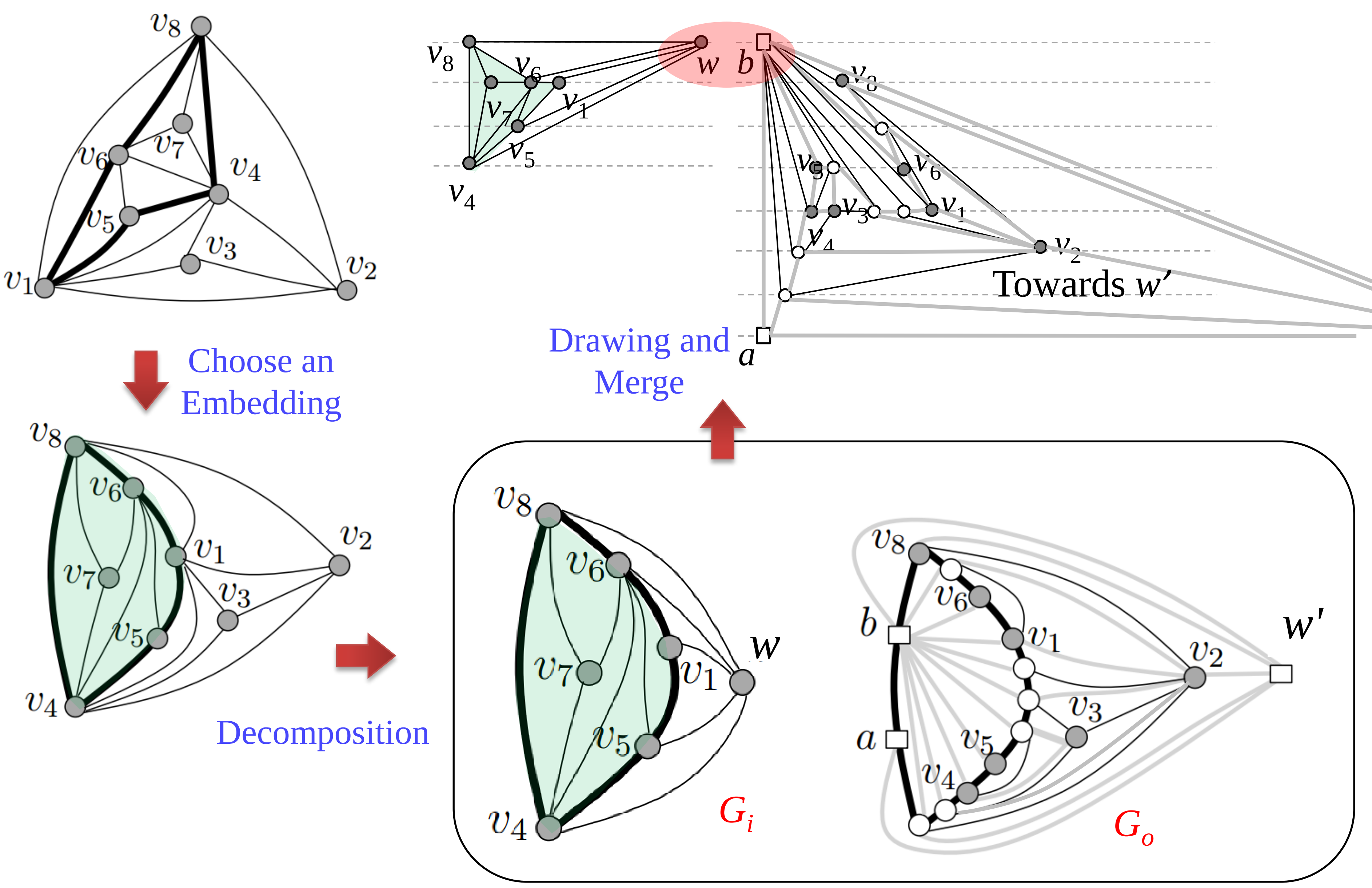


A separator
of size $O(\sqrt{n})$

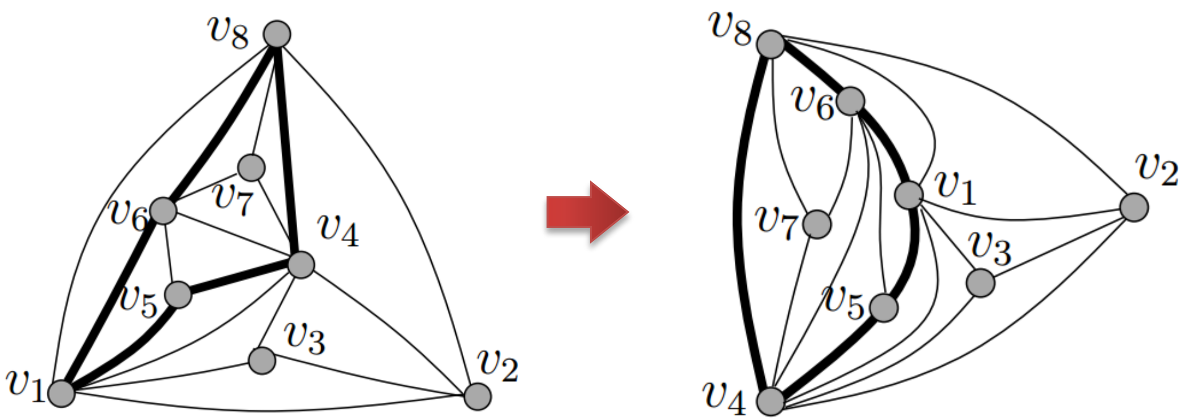


G_i with $2n/3+O(\sqrt{n})$ vertices

The Big Picture

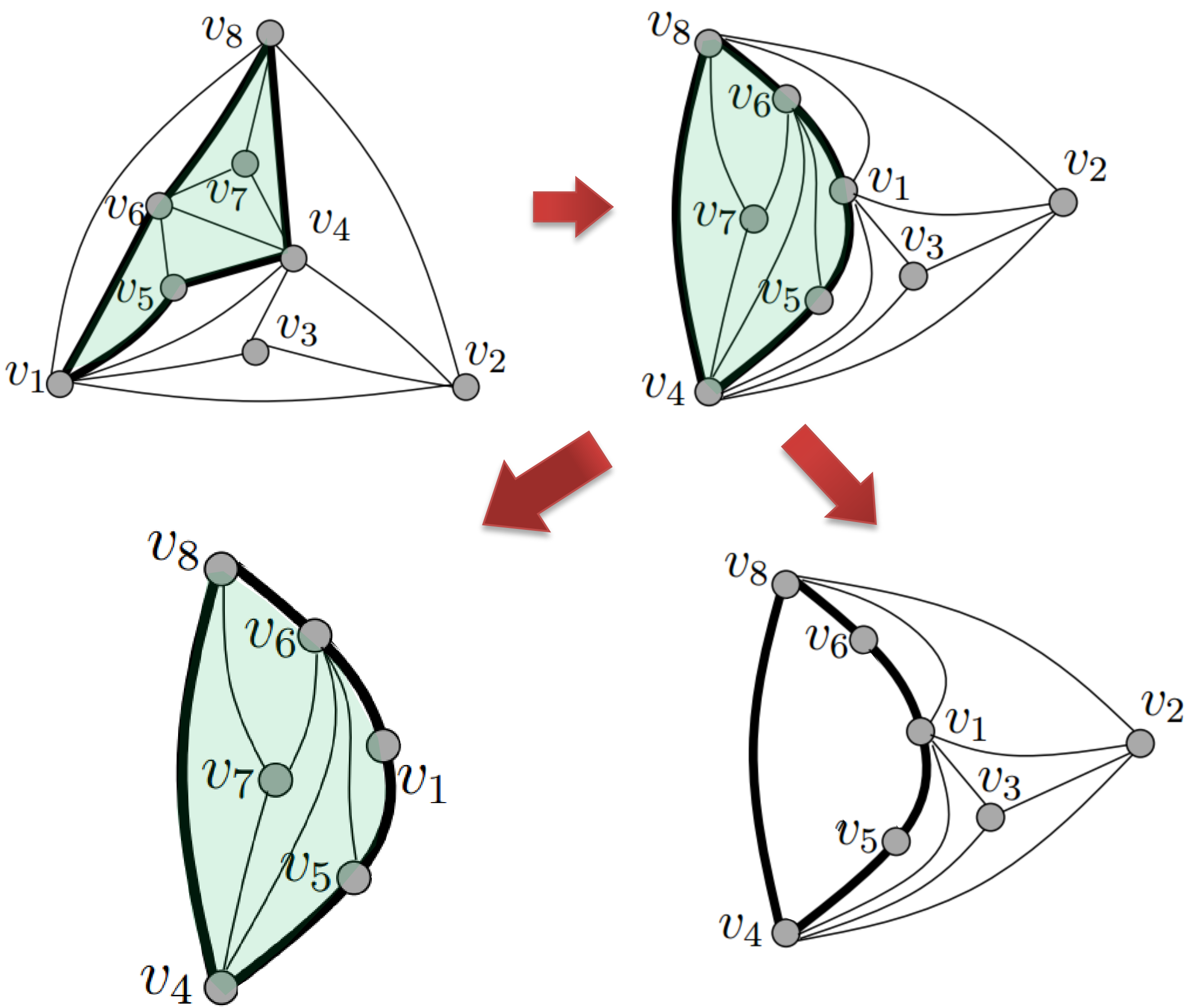


Technical Details (Choose an Embedding)



Choose a face which is incident to some edge of the cycle separator as the new outer face.

Technical Details (Construct G_o and G_i)

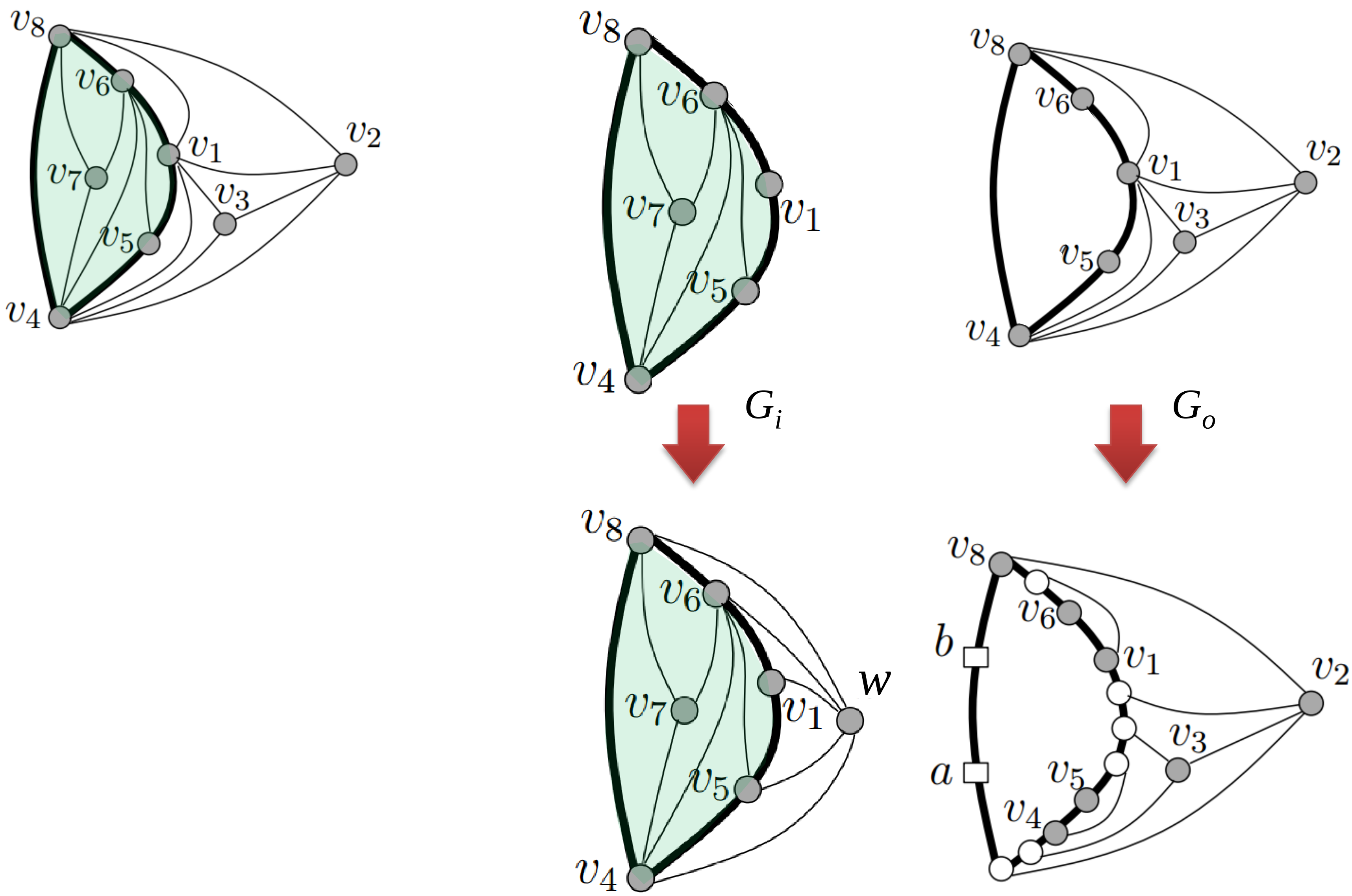


Choose a face which is incident to some edge of the cycle separator as the new outer face.

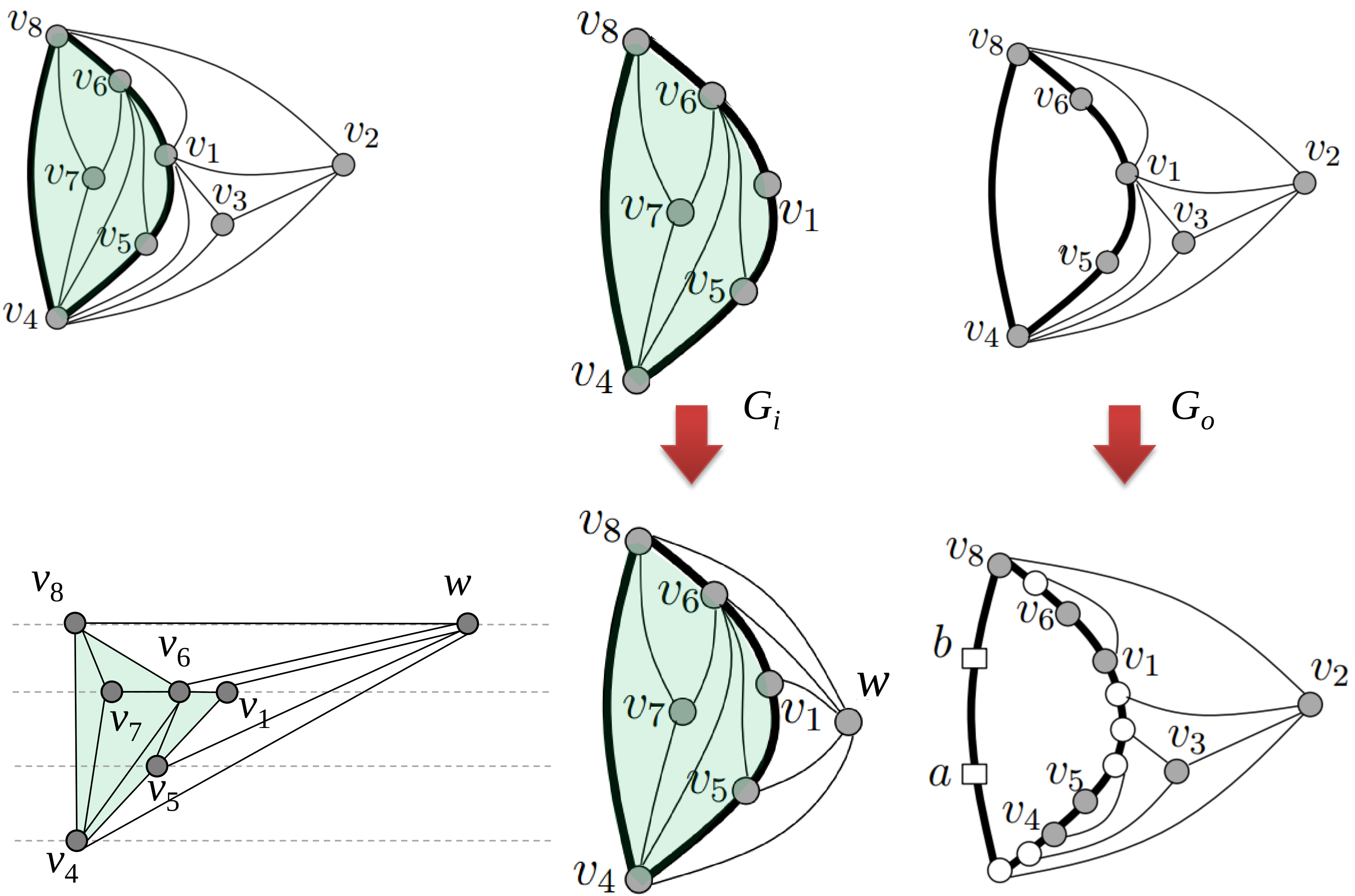
Construct G_o and G_i

G_i G_o

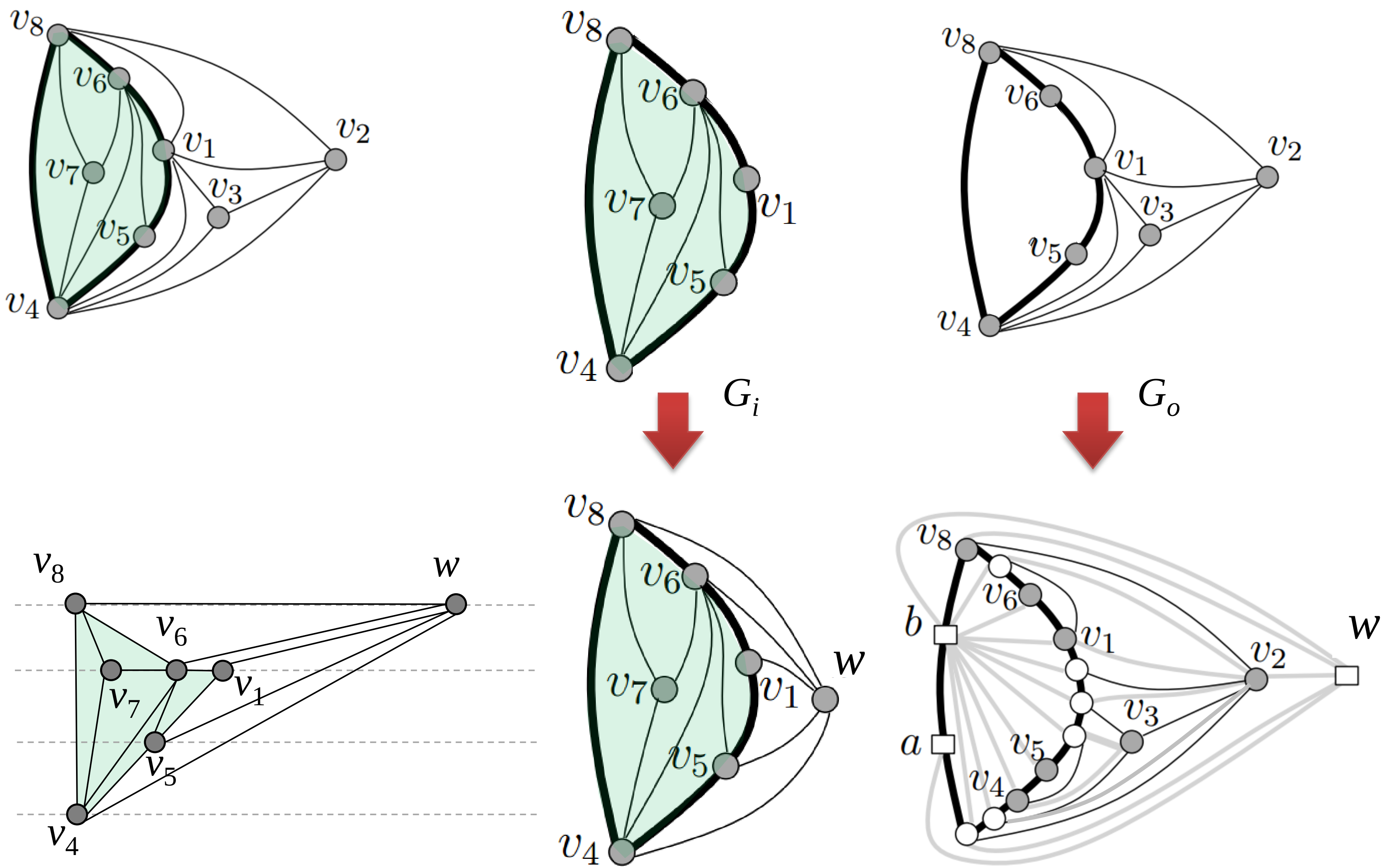
Technical Details (Draw G_o and G_i)



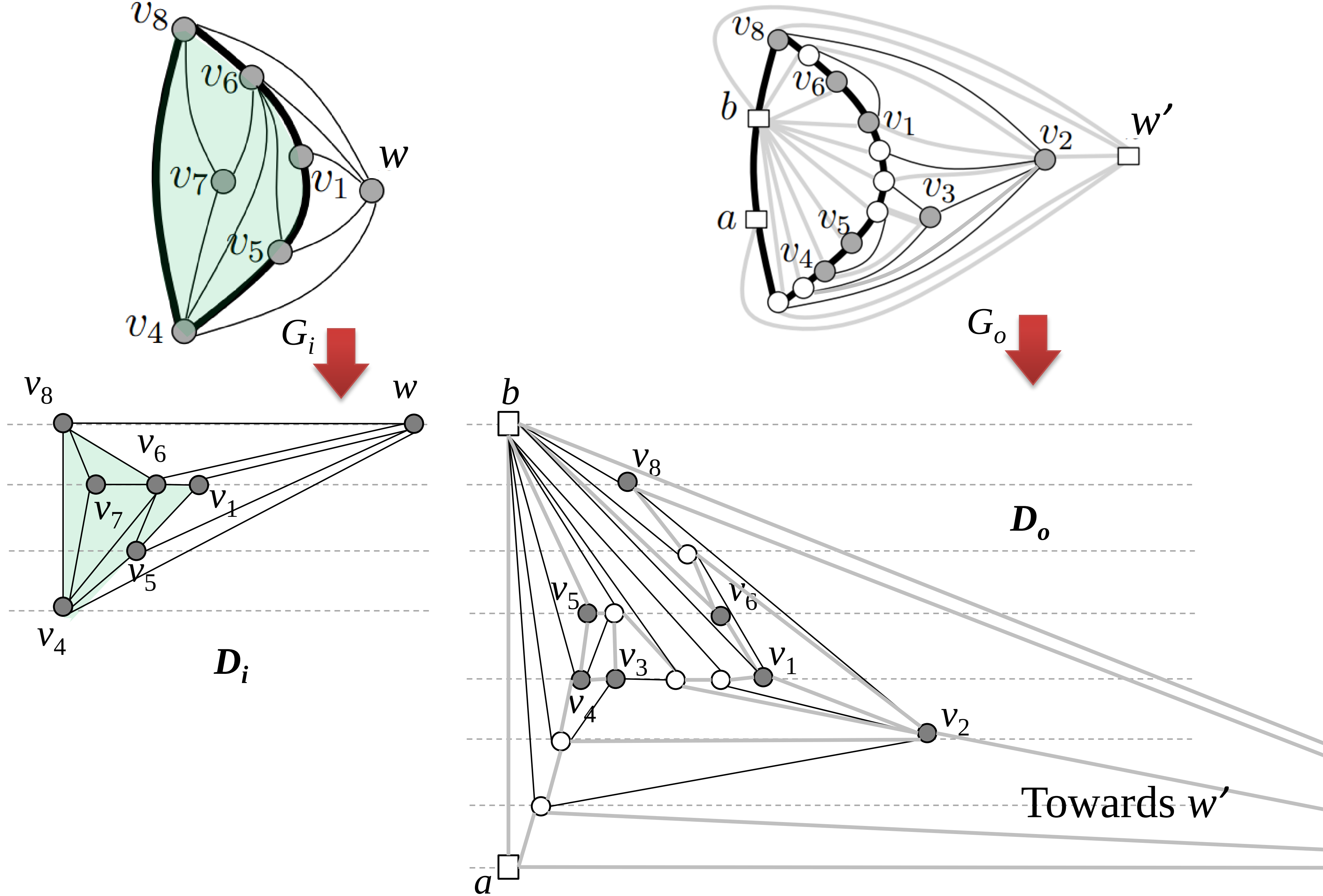
Technical Details (Draw G_o and G_i)



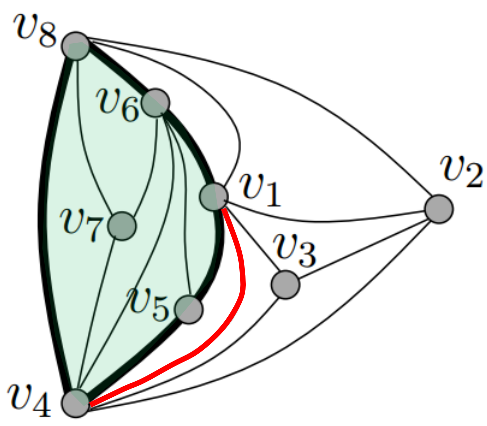
Technical Details (Draw G_o and G_i)

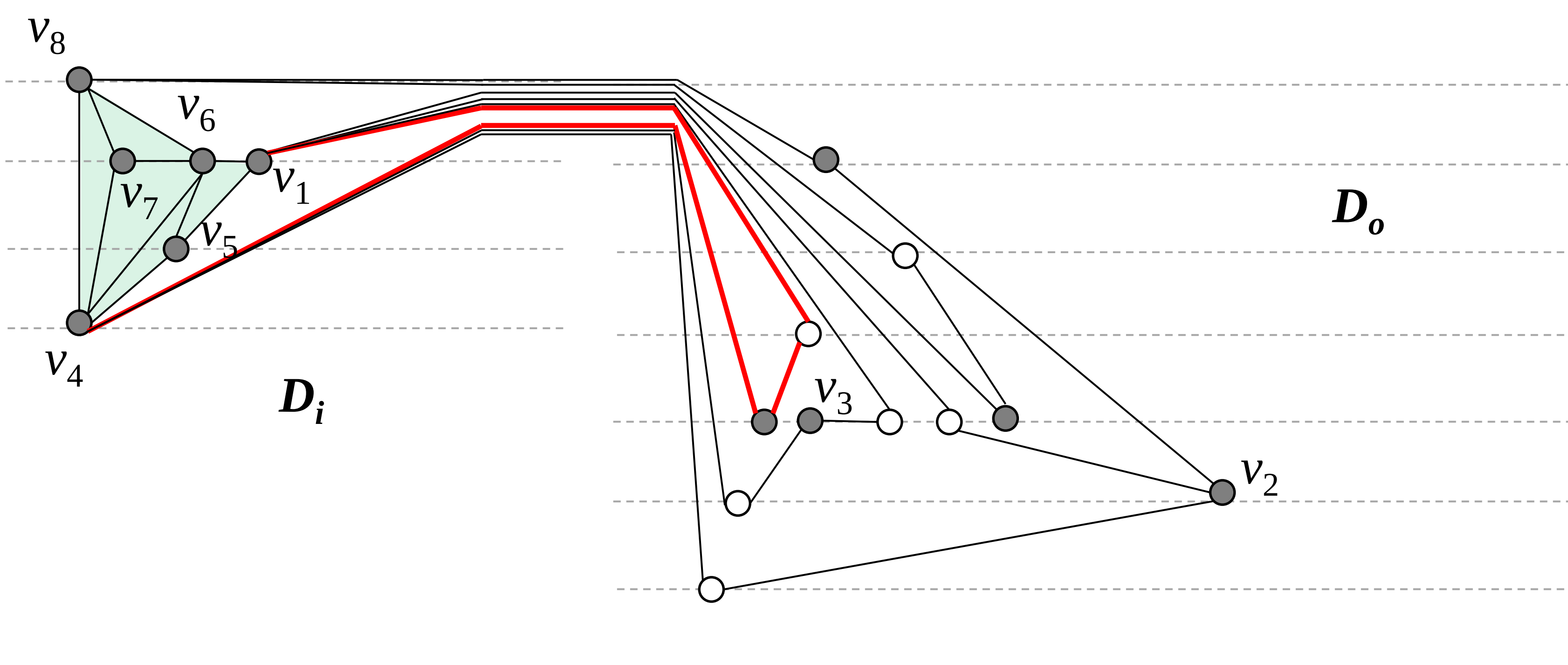


Technical Details (Draw G_o and G_i)

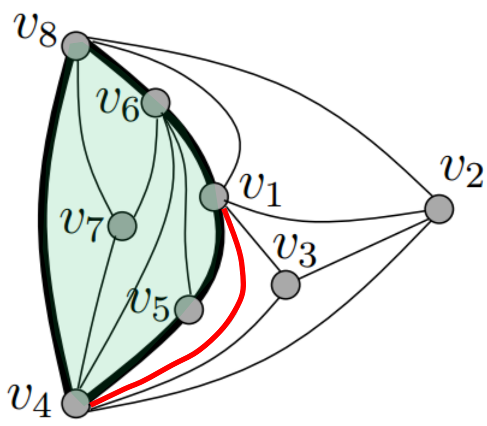


Technical Details (Merge D_o and D_i)





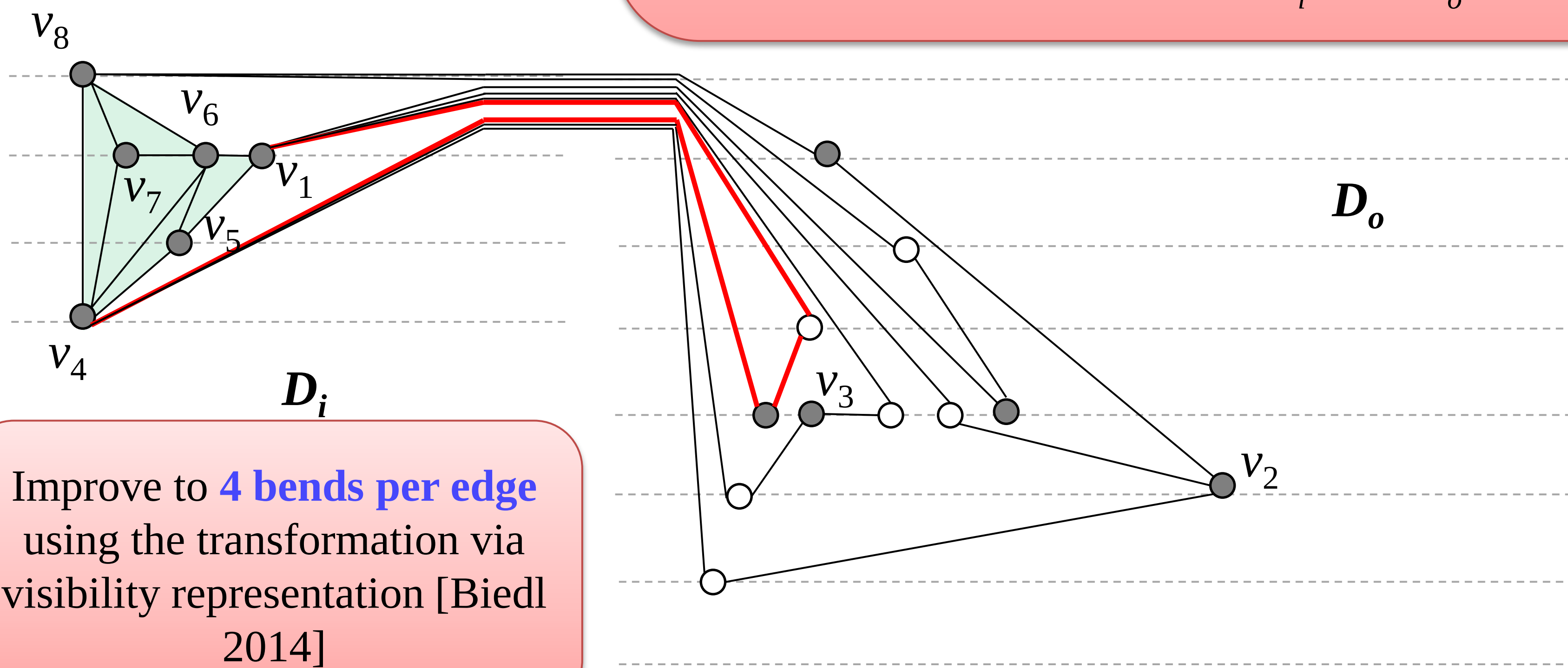
Technical Details (Merge D_o and D_i)



Height of D_i is $(2/3)\times|D_i| = 4n/9+O(\lambda)$
Height of D_o is $(2/3)\times|D_o| = 4n/9+O(\lambda\Delta)$
Height of the final drawing is **$4n/9+O(\lambda\Delta)$**

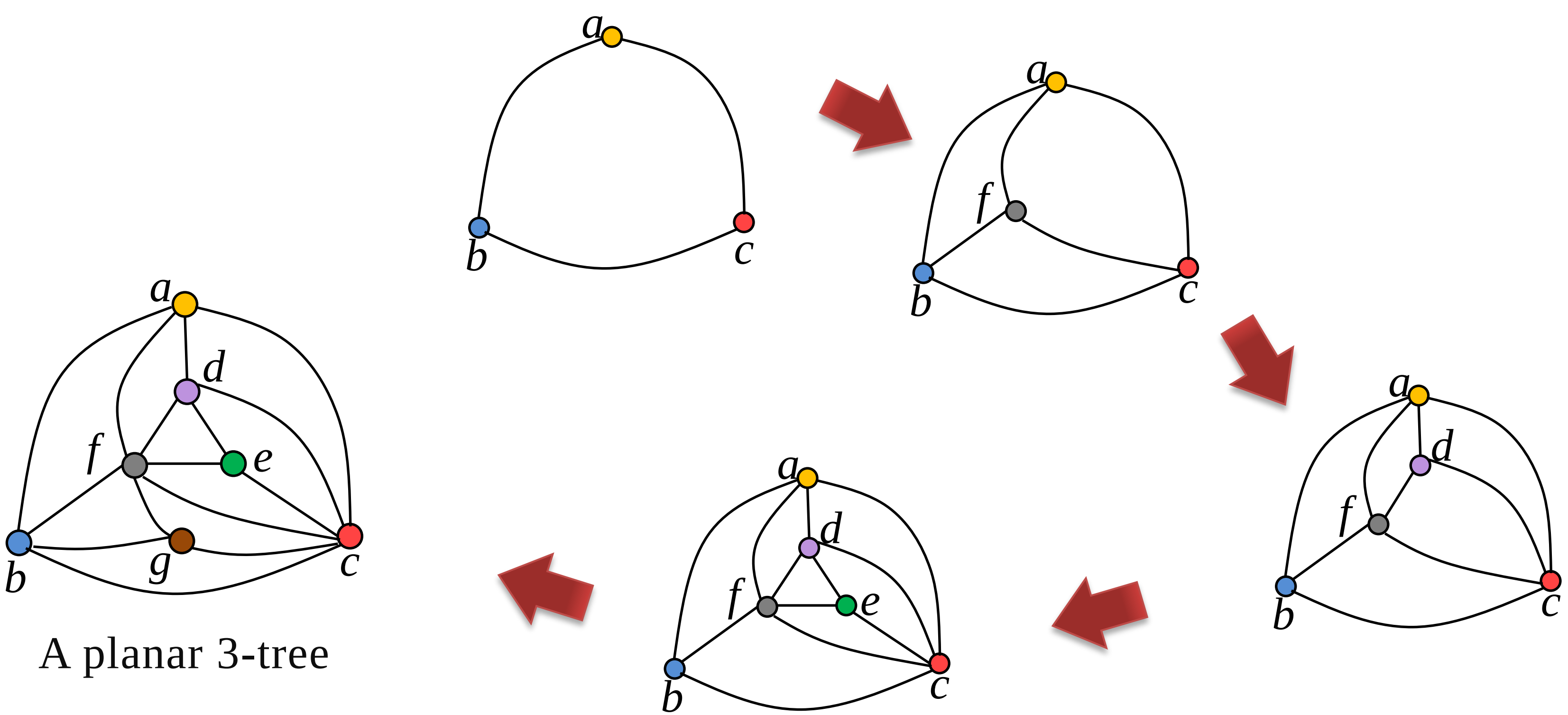
At most **6 bends per edge**

- two bends to enter D_o from D_i
- two bends on separator
- two bends to return to D_i from D_o



Improve to **4 bends per edge** using the transformation via visibility representation [Biedl 2014]

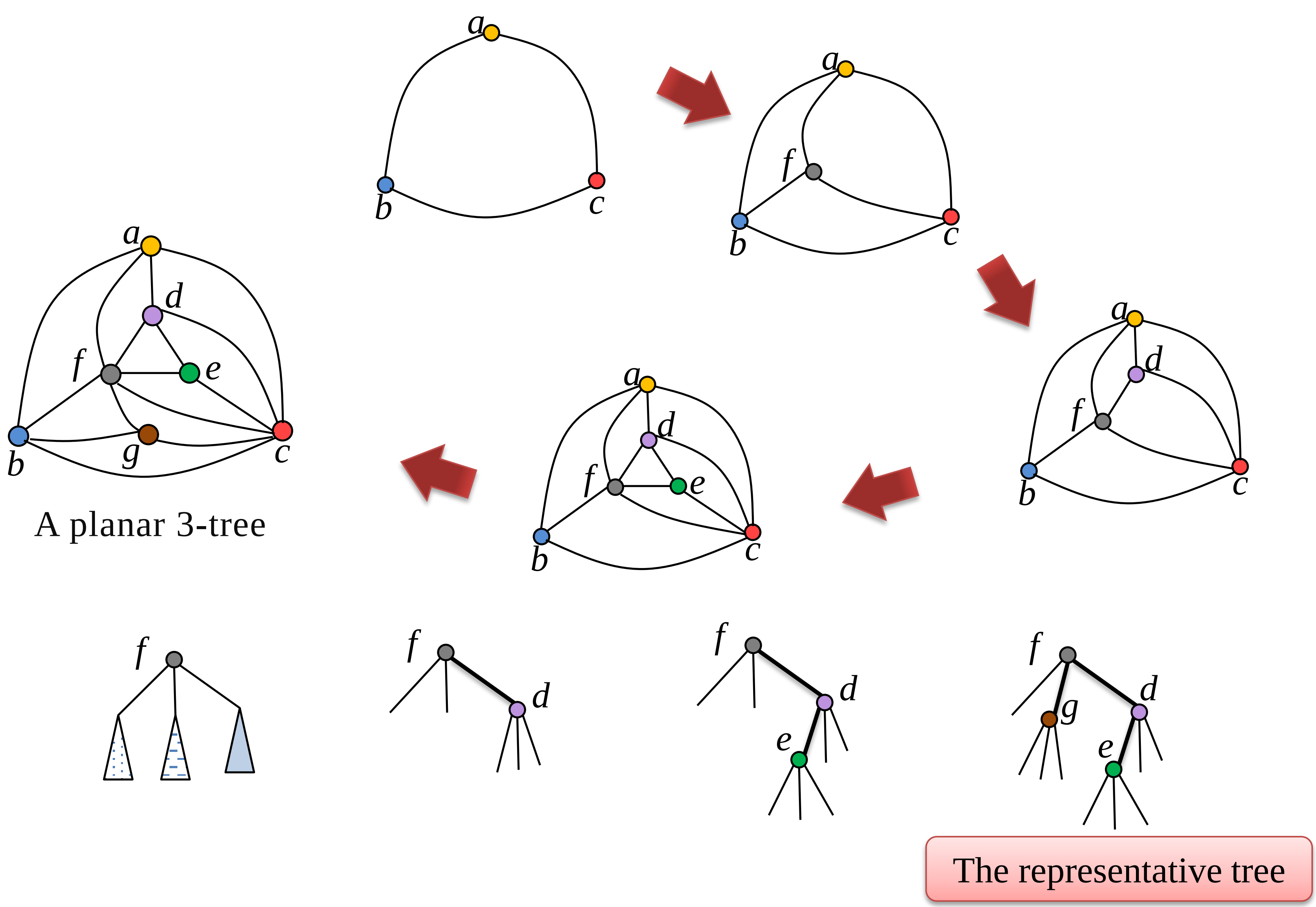
Plane 3-Trees



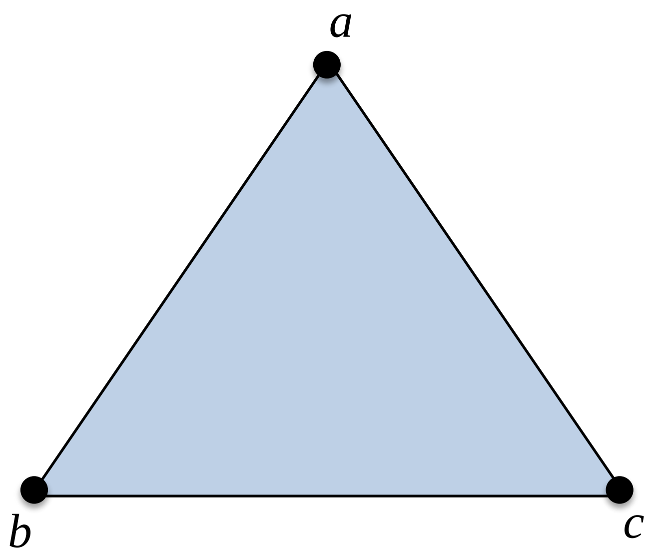
A planar 3-tree

Start with a triangle, then repeatedly add a vertex and triangulate the resulting graph.

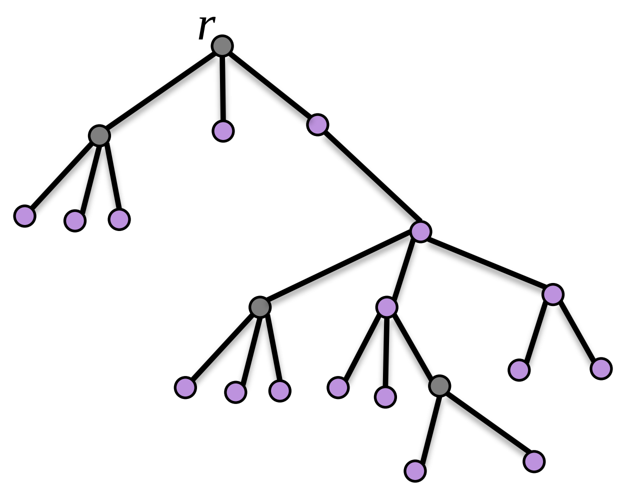
Plane 3-Trees



Plane 3-Trees

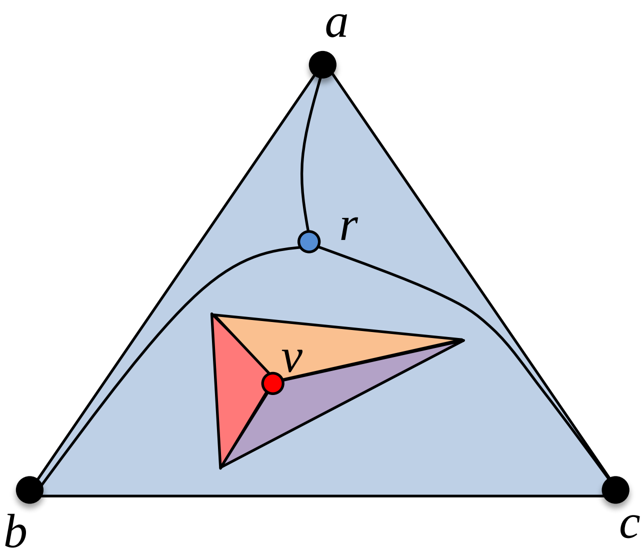


A planar 3-tree *G*

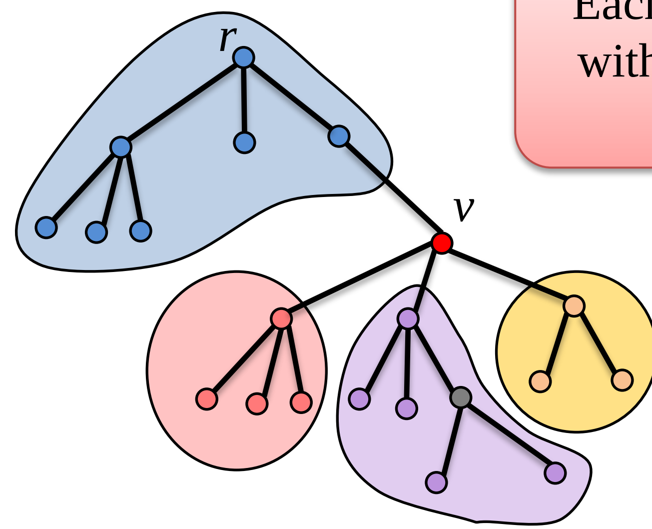


The representative tree *T* of *G*

Plane 3-Trees

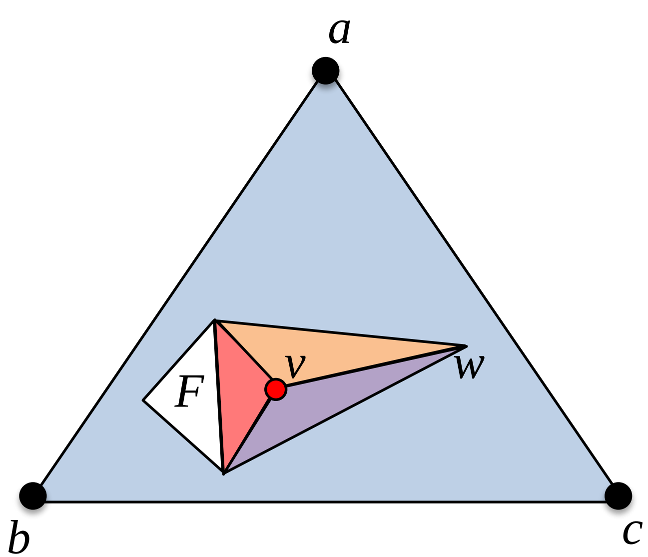


A planar 3-tree G

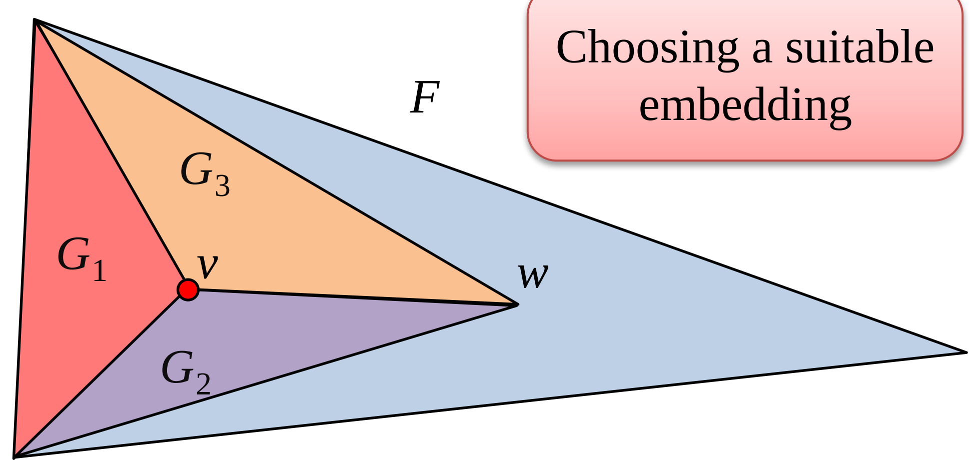


The representative tree T of G

Each component with at most $n/2$ vertices

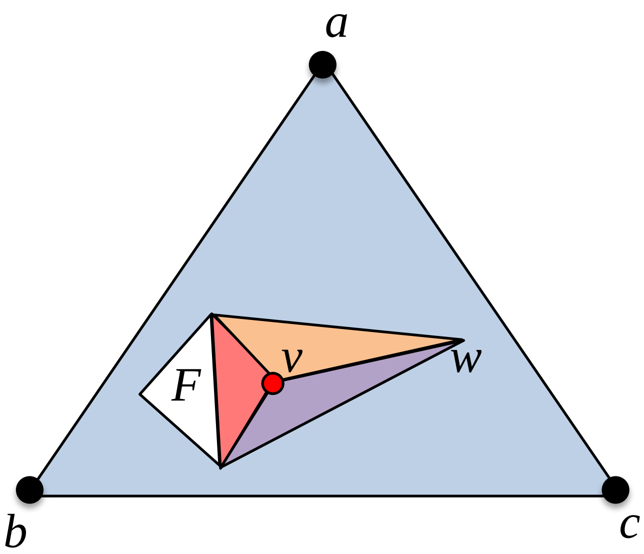


A planar 3-tree G

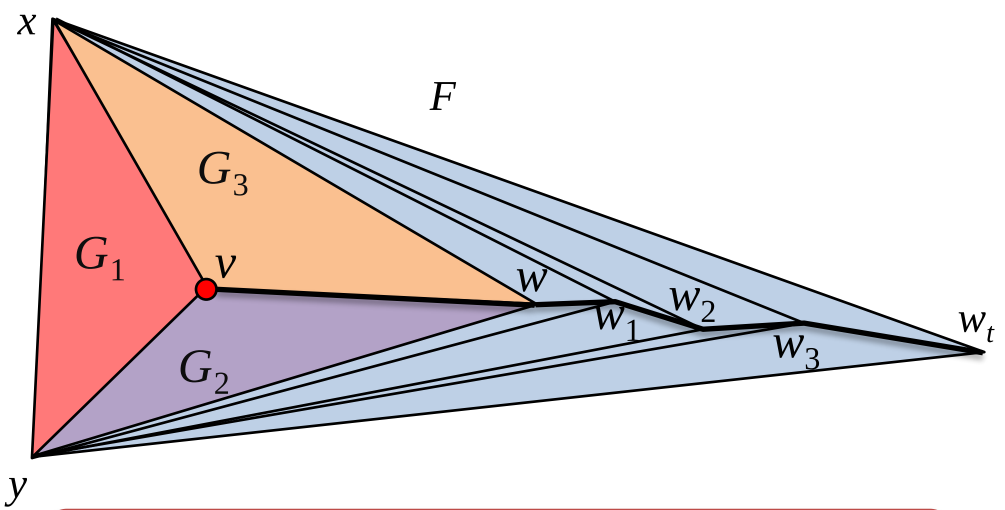


Choosing a suitable embedding

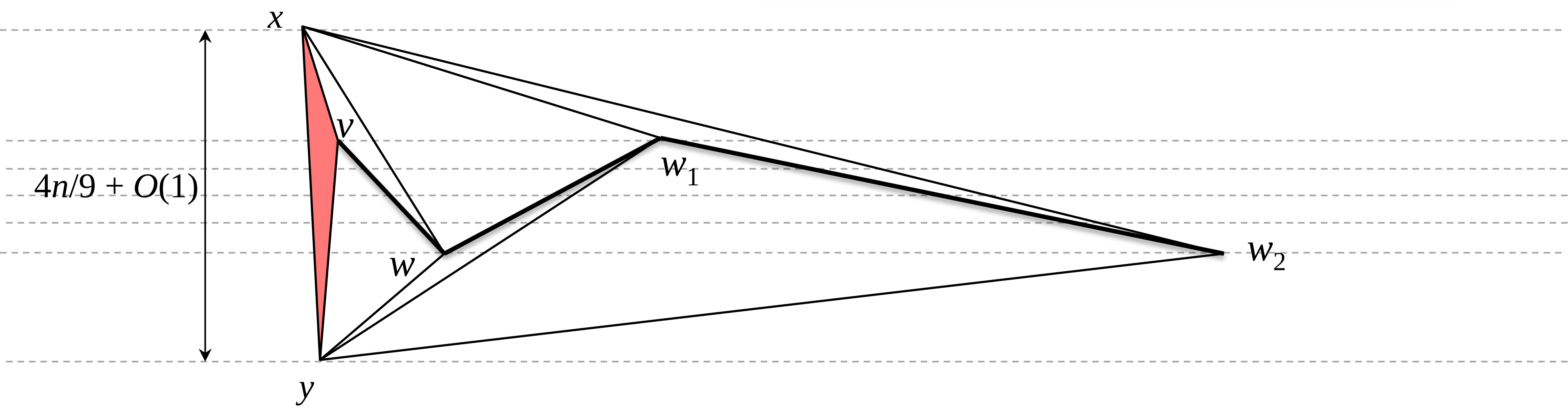
Plane 3-Trees



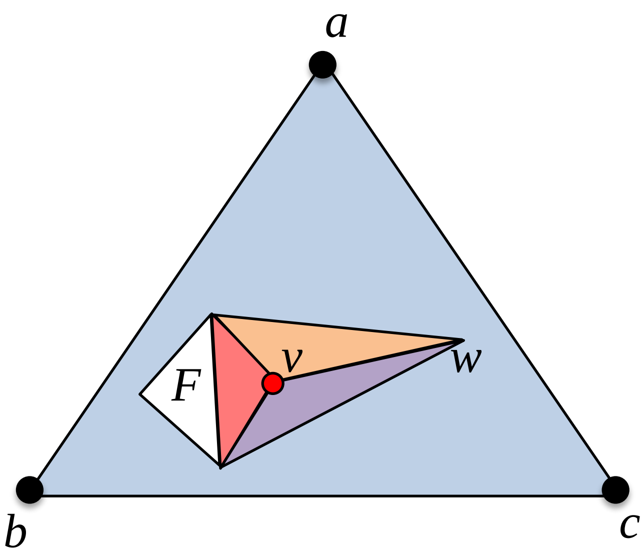
A planar 3-tree G



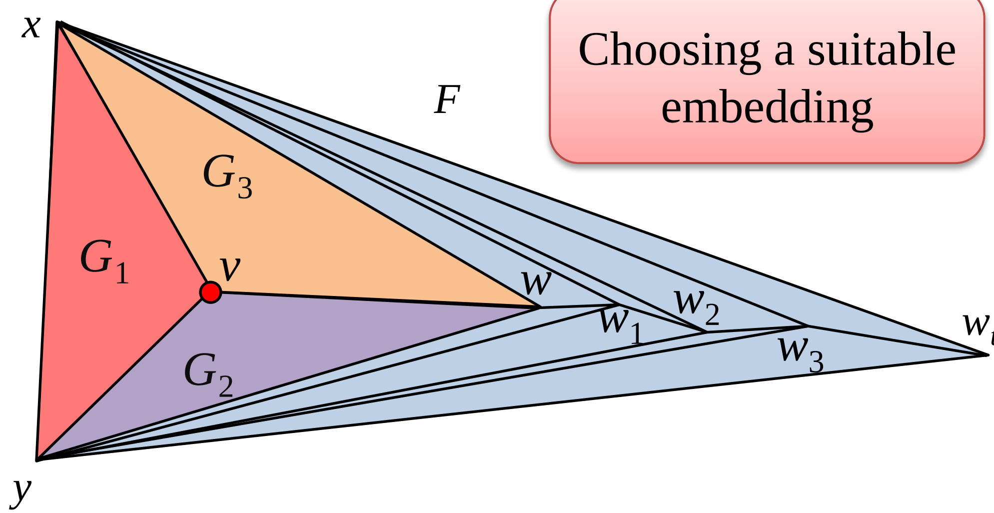
Plane 3 trees inside each of these triangles has $n/2+O(1)$ vertices



Plane 3-Trees

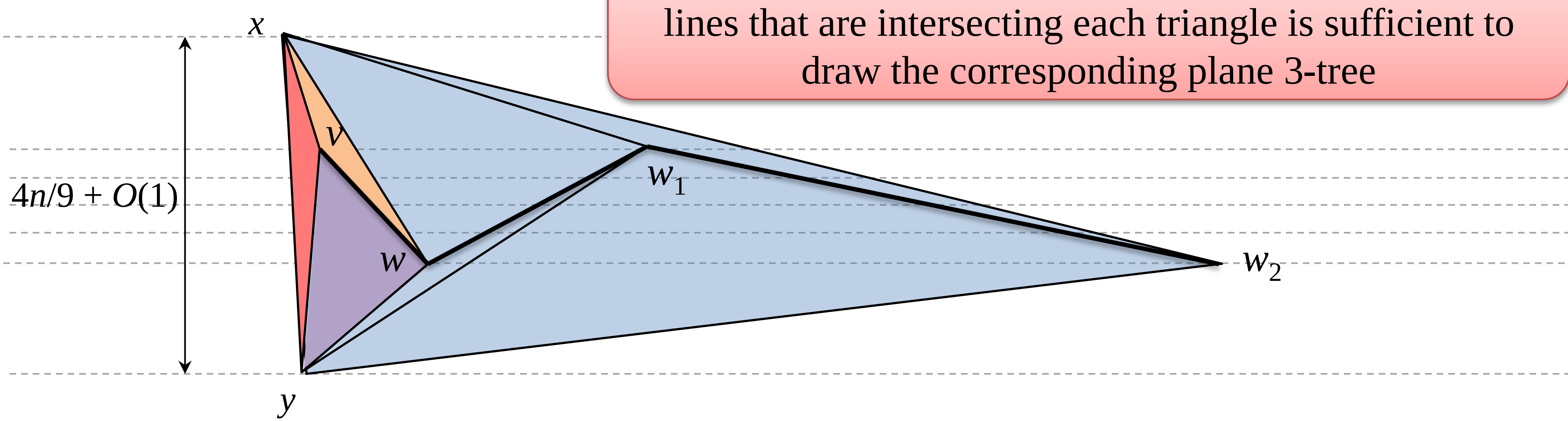


A planar 3-tree G



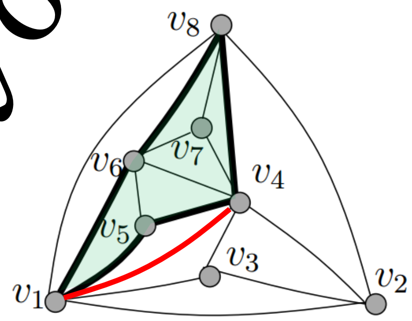
Choosing a suitable embedding

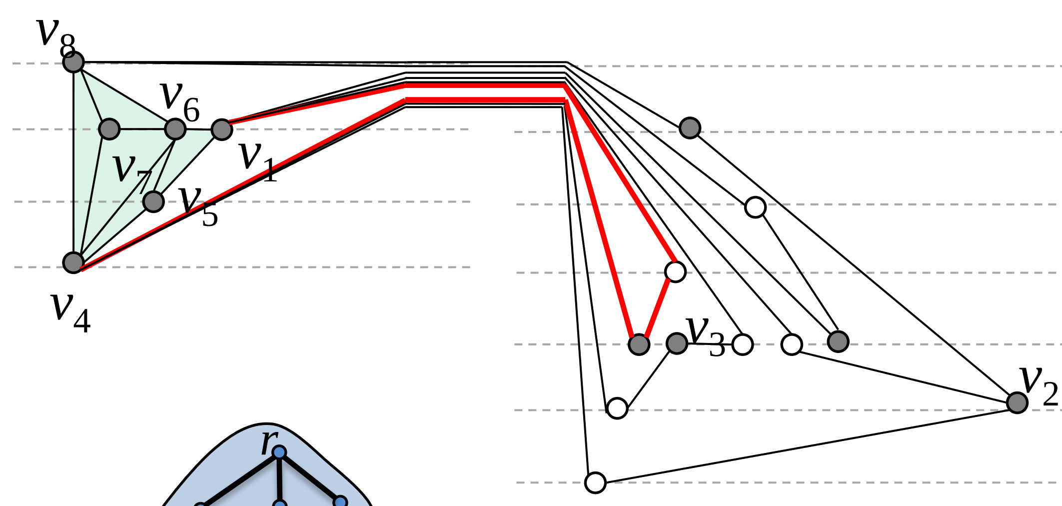
The main challenge here is to show that the number of lines that are intersecting each triangle is sufficient to draw the corresponding plane 3-tree

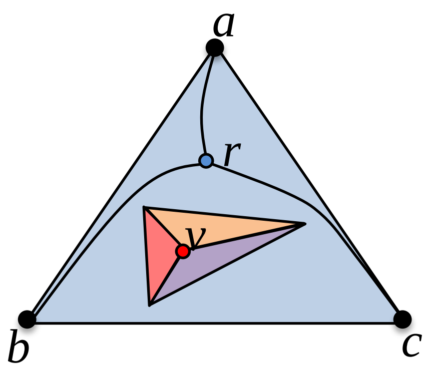


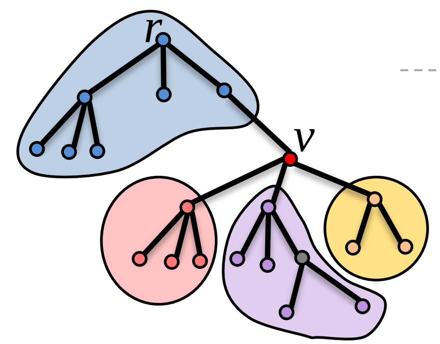
Thank you

OPEN: Close the gap!









Upper Bounds

Triangulations

Area	Height		
$0.88n^2 + O(1)$	$0.66n$		[Brandenburg 2008]
$0.44n^2 + O(1)$	$0.66n$	(polyline)	[Bonichon et al. 2003]

Planar 3-trees

Area	Height	
$0.88n^2 + O(1)$	$0.5n$	[Brandenburg 2008, Hossain et al. 2013]

Improved Upper Bounds

Triangulations

Polyline drawing with height

$4n/9+O(\lambda\Delta) \approx 0.44n+O(\lambda\Delta)$

This is $0.44n+o(n)$ when Δ is $o(n)$

Planar 3-trees

Straight-line drawing with height

$4n/9+O(1) \approx 0.44n+O(1)$