Trade-offs in Planar Polyline Drawings

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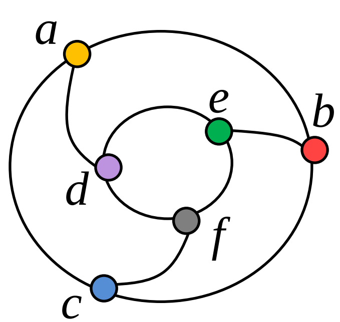
Aesthetics of Polyline Drawings



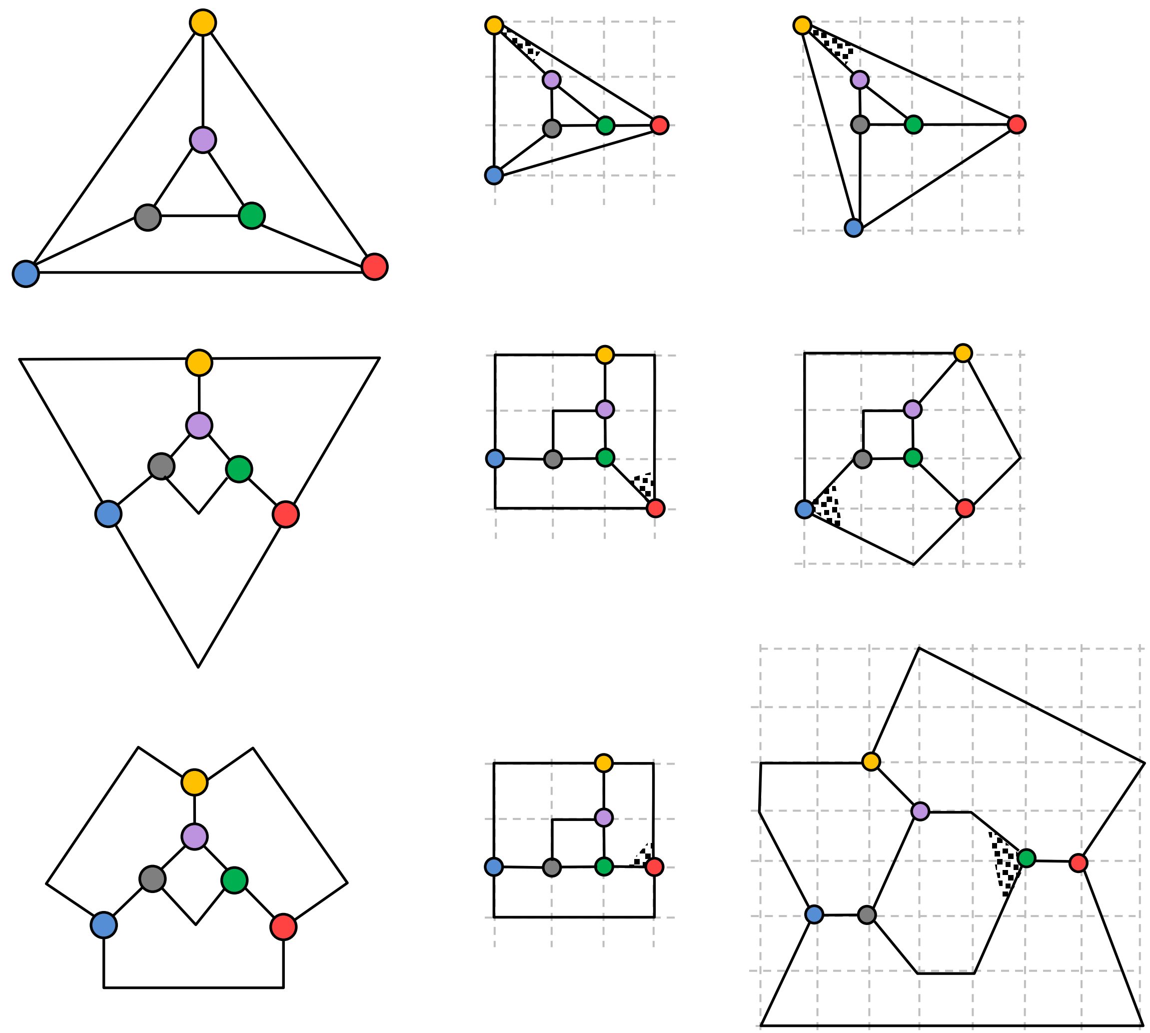
A planar graph G

k-bend drawings of G, with angular resolution θ and area A

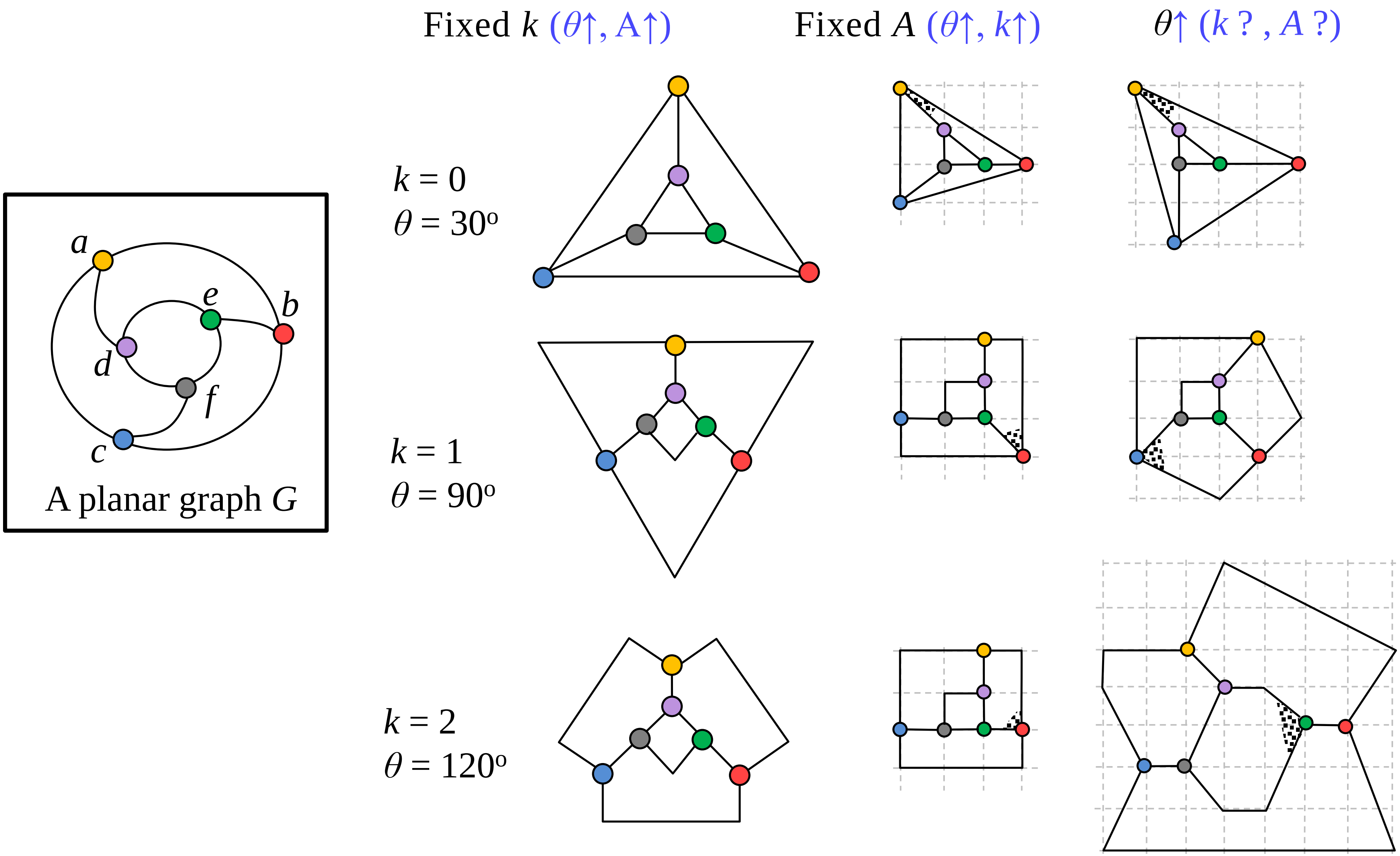
$k = 0$
$\theta = 30^{o}$

$k = 1$
$\theta = 90^{o}$

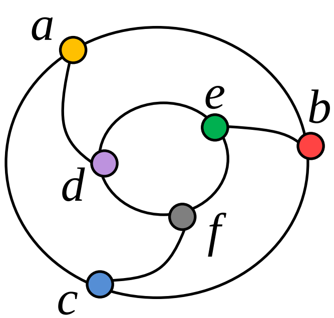
$k = 2$
$\theta = 120^{o}$



Expected Trade-offs ?



Expected Trade-offs ?



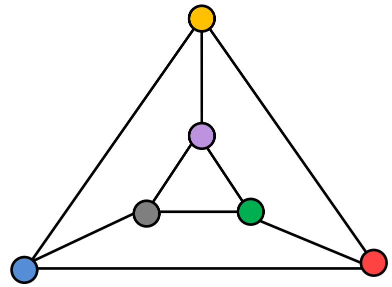
A planar graph G

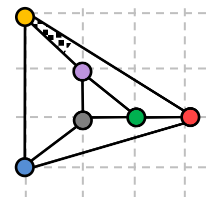
Fixed k (θ↑, A↑)

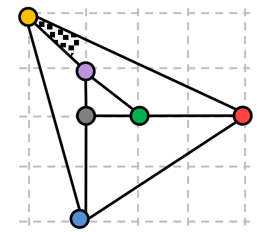
Fixed A (θ↑, k↑)

θ↑ (k ? , A ?)

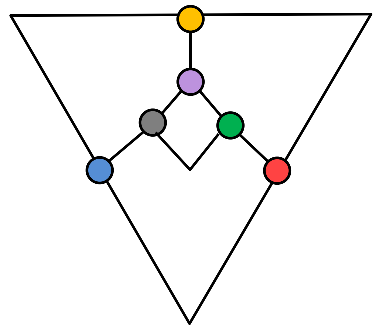
$k = 0$
$\theta = 30^o$

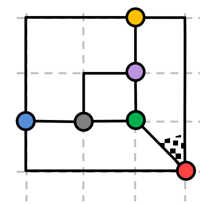


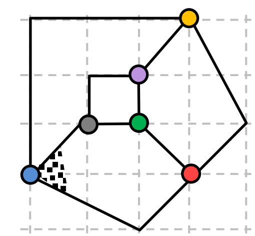




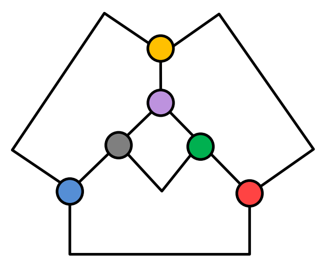
$k = 1$
$\theta = 90^o$

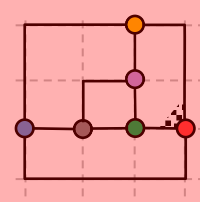


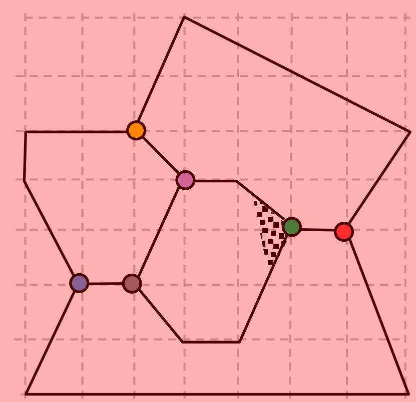




$k = 2$
$\theta = 120^o$







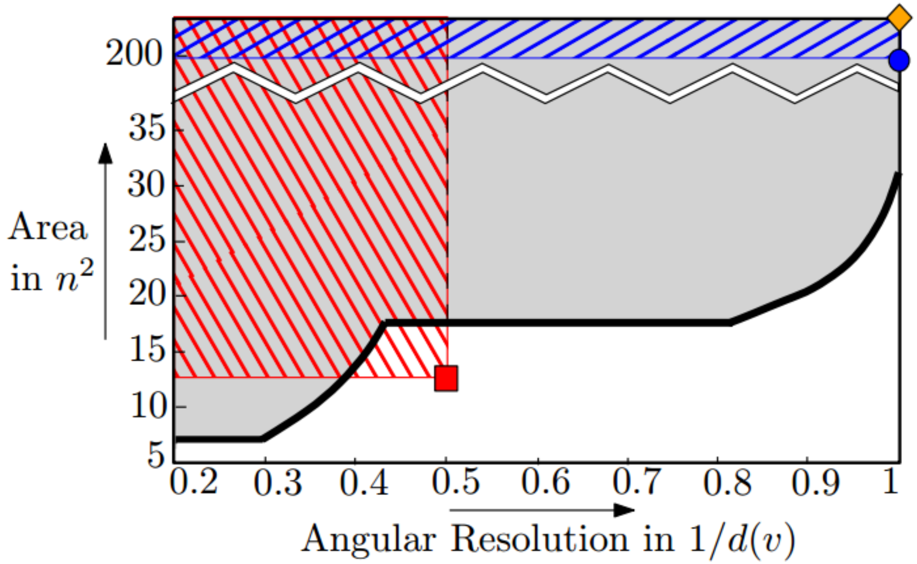
Previous Results (on Triangulations)

***k*-Bend Drawings**	**Area**	**Angular R.**	**Total Bends**	**Reference**
$k = 0$	$0.89n^2$	$\Omega(1/n^2)$	0	Brandenburg, ENDM 2008
$k = 0$	$4.50n^2$	$\Omega(1/n)$	0	Kurowski, SOFSEM 2005
$k = 0$	$\Omega(c^{\rho n})$	$\Omega(1/\rho)$	0	Garg and Tamassia, IEEE SMC 1988
$k = 1$	$0.45n^2$	$\Omega(1/n^2)$	$2n/3$	Zhang, Algorithmica 2010
$k = 1$	$12.5n^2$	$\Omega(0.5/d_v)$	$3n$	Duncan and Kobourov, JGAA 2003
$k = 1$	$450n^2$	$\Omega(1/d_v)$	$3n$	Cheng, Duncan, Goodrich, Kobourov, DCG 2001
$k = 2$	$200n^2$	$\Omega(1/d_v)$	$6n$	Goodrich and Wagner, Algorithms 2000
$\mathbf{k = 2}$	$\mathbf{(6\alpha+8/3)^2 n^2}$	$\mathbf{\frac{\alpha}{d_v(\alpha^2+1/4)}}$	$\mathbf{5.5n}$	**This Presentation**
$\mathbf{k = 2}$	$\mathbf{(6\beta + 2/3)^2 n^2}$	$\mathbf{\frac{\beta}{d_v(\beta^2+1)}}$	$\mathbf{5.5n}$	**This Presentation**

(A↑, θ↑)

(A↑, θ↑)

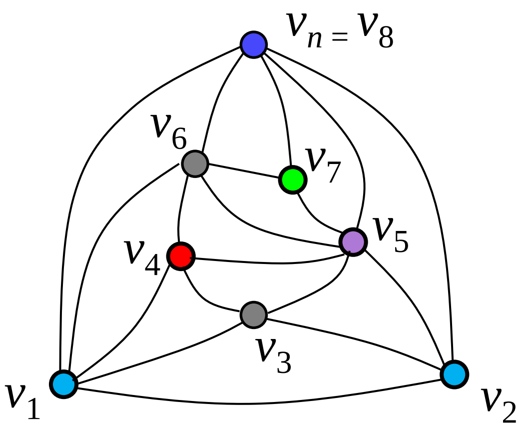
The Big Picture



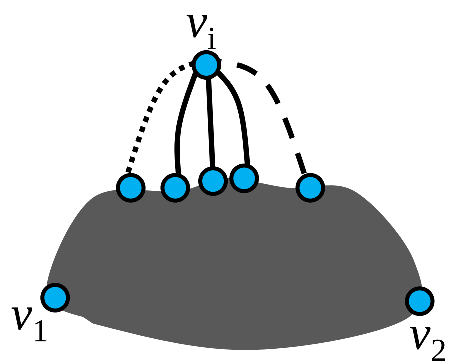
	k-Bend Drawings	Area	Angular R.	Total Bends	Reference
■	$k = 1$	$12.5n^2$	$\Omega(0.5/d_v)$	$3n$	Duncan and Kobourov, JGAA 2003
◆	$k = 1$	$450n^2$	$\Omega(1/d_v)$	$3n$	Cheng, Duncan, Goodrich, Kobourov, DCG 2001
●	$k = 2$	$200n^2$	$\Omega(1/d_v)$	$6n$	Goodrich and Wagner, Algorithms 2000
	$k = 2$	$(6\alpha+8/3)^2 n^2$	$\frac{\alpha}{d_v(\alpha^2+1/4)}$	$5.5n$	**This Presentation**
	$k = 2$	$(6\beta + 2/3)^2 n^2$	$\frac{\beta}{d_v(\beta^2+1)}$	$5.5n$	**This Presentation**

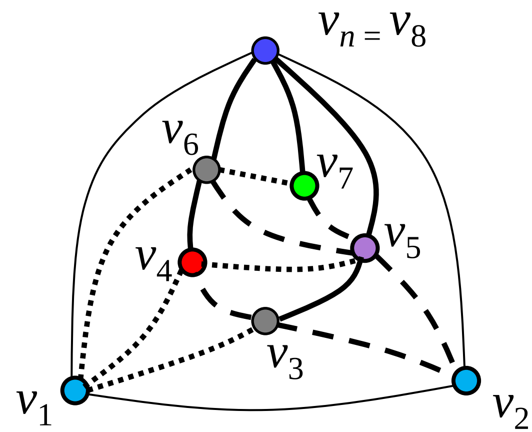
Tools and Techniques

Idea: Decomposition of a Triangulation into Trees + L-Contact Representation

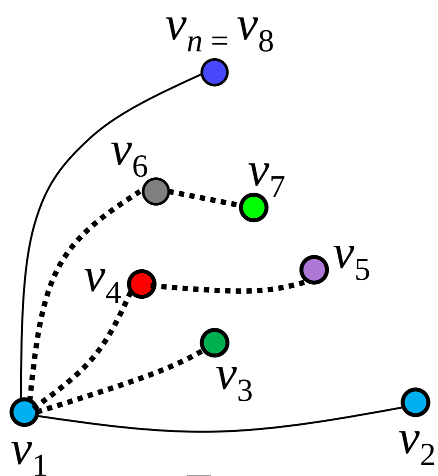


A Canonical Ordering of G
[De Fraysseix et al. 1988]

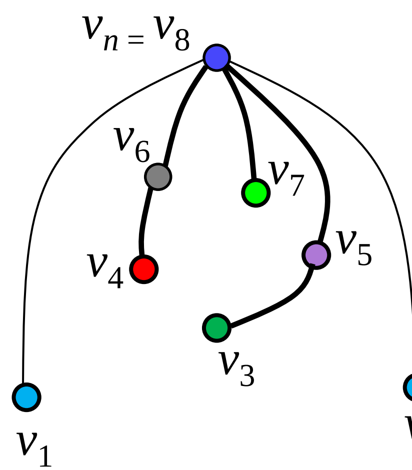




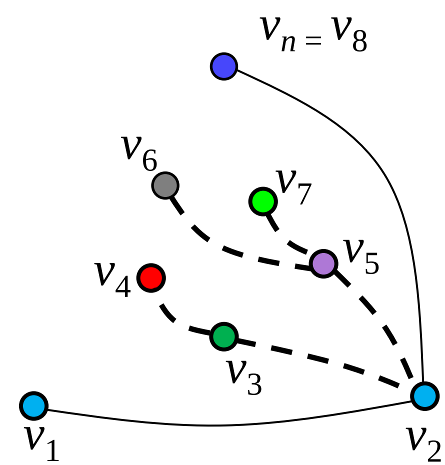
A Schnyder realizer of G
[Schnyder 1990]



T_{left}



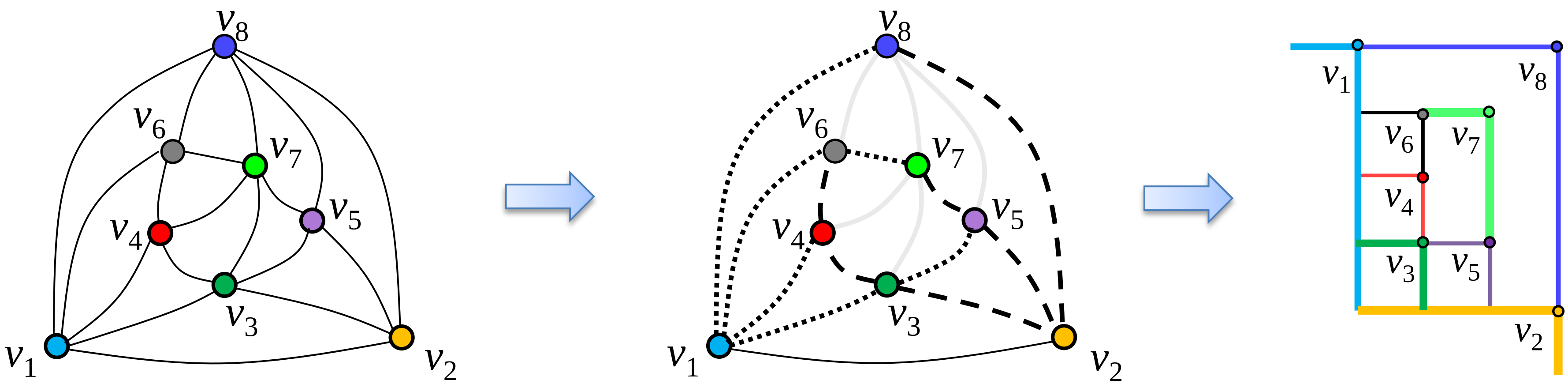
T_{mid}

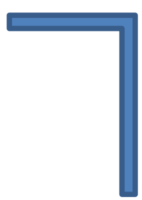


T_{right}

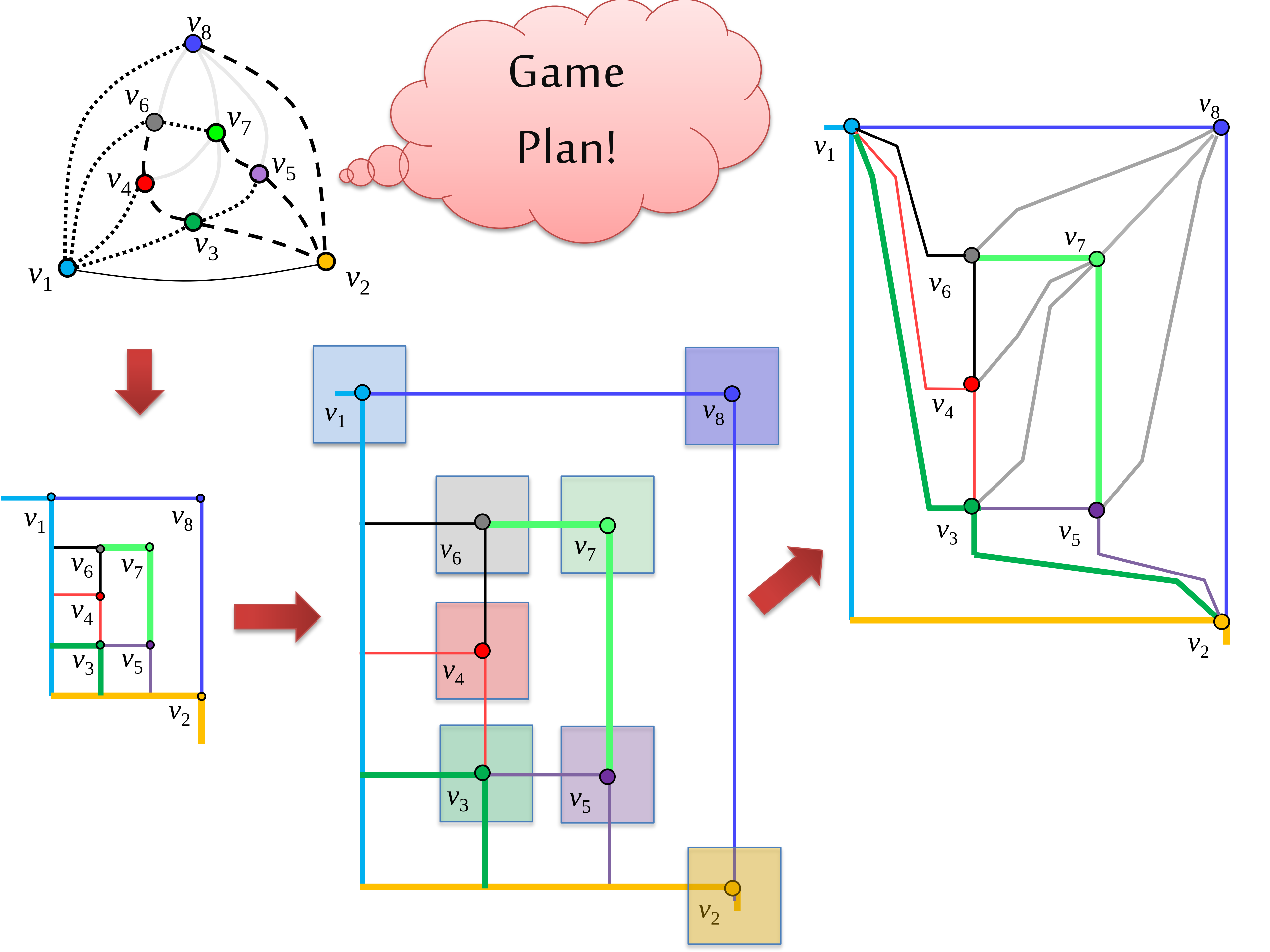
Tools and Techniques

Idea: Decomposition of a Triangulation into Trees + *L*-Contact Representation





Vertices are represented with *L*-shapes
(flipped horizontally)



Game
Plan!
v_8
v_6
v_7
v_5
v_4
v_3
v_1
v_2
v_1
v_8
v_6
v_7
v_4
v_3
v_5
v_2
v_1
v_8
v_6
v_7
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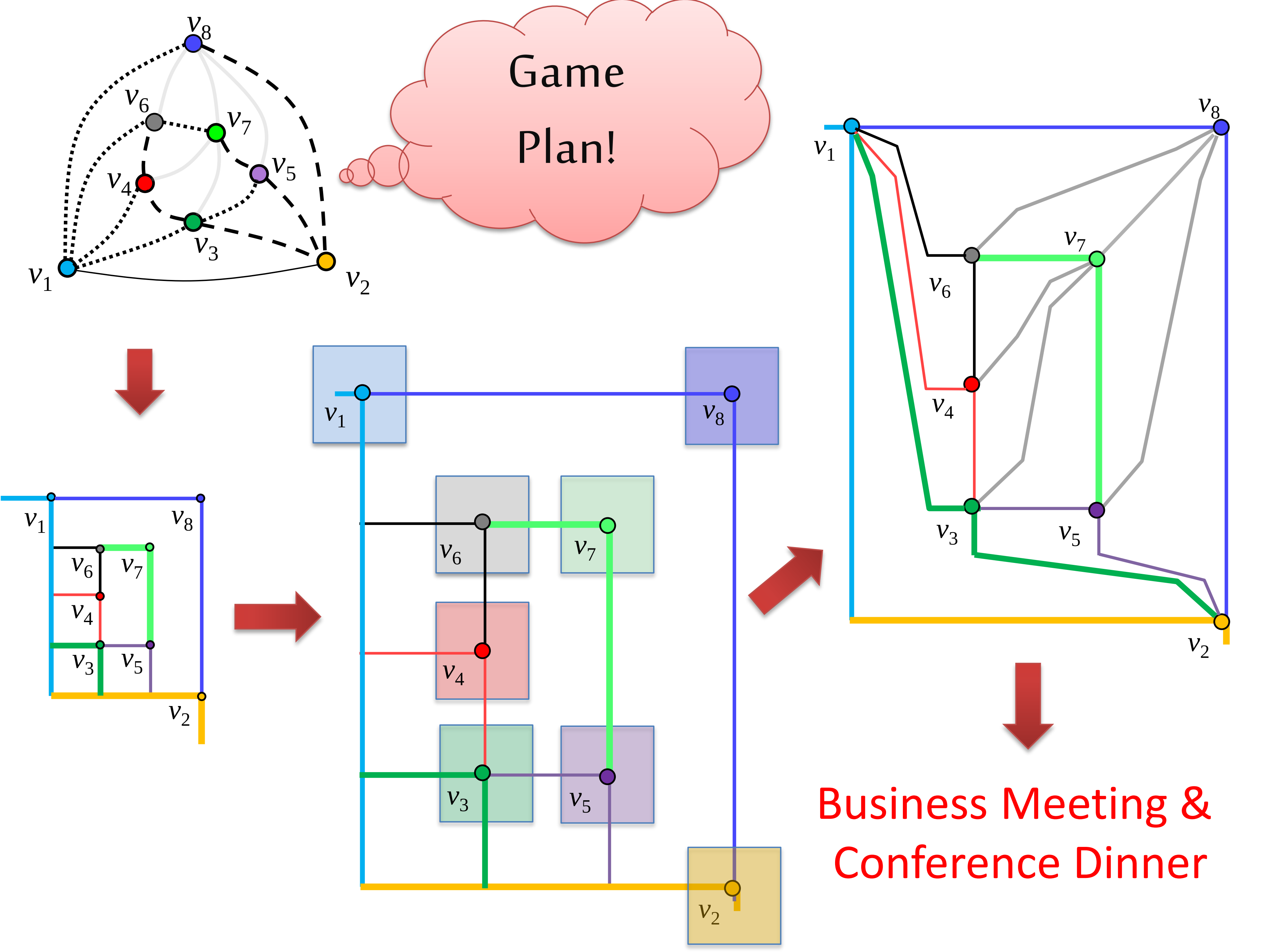
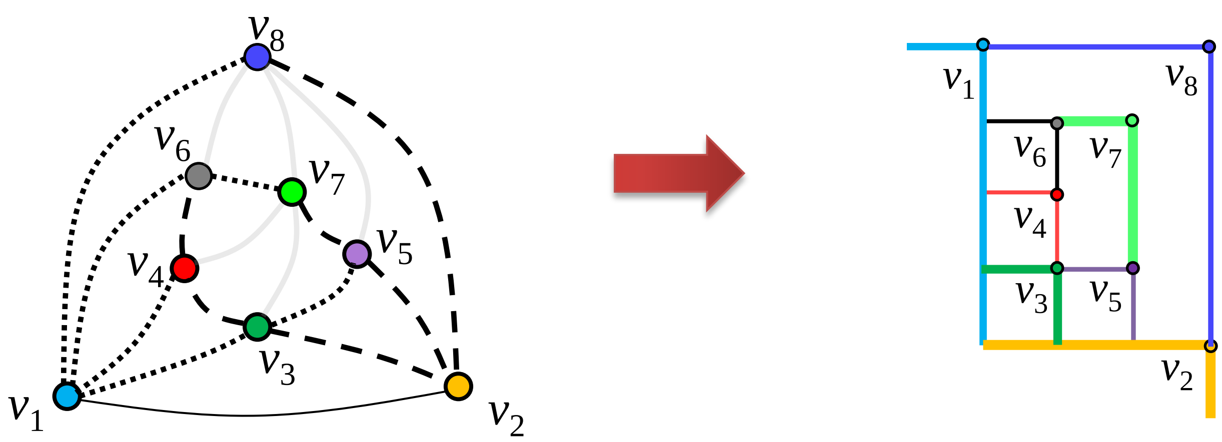
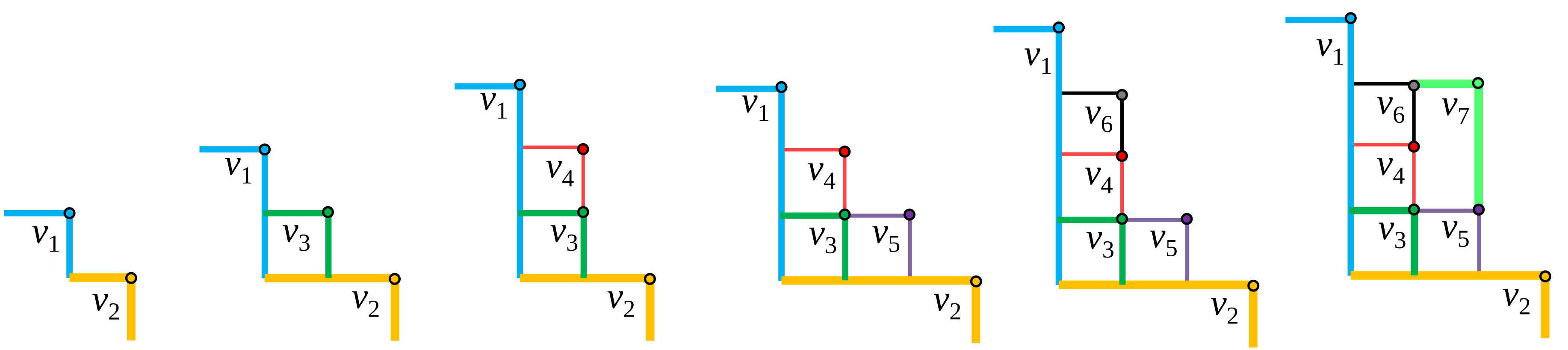
v_4
v_3
v_5
v_2

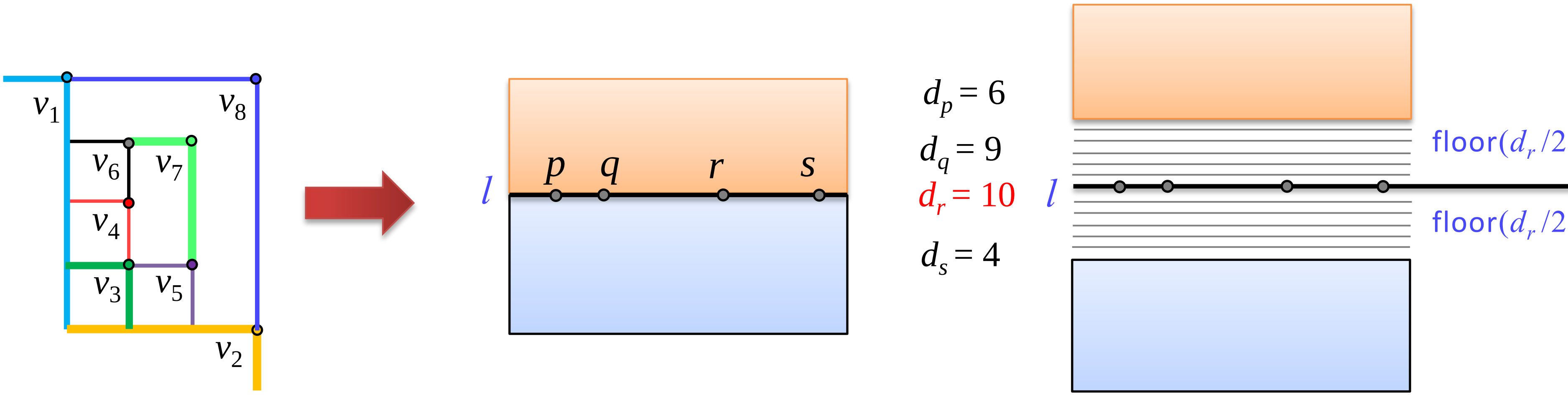

Game
Plan!
v8
v6
v7
v5
v4
v3
v1
v2
v1
v8
v6
v7
v4
v3
v5
v2
Business Meeting &
Conference Dinner

Phase 1 (Contact Representation)





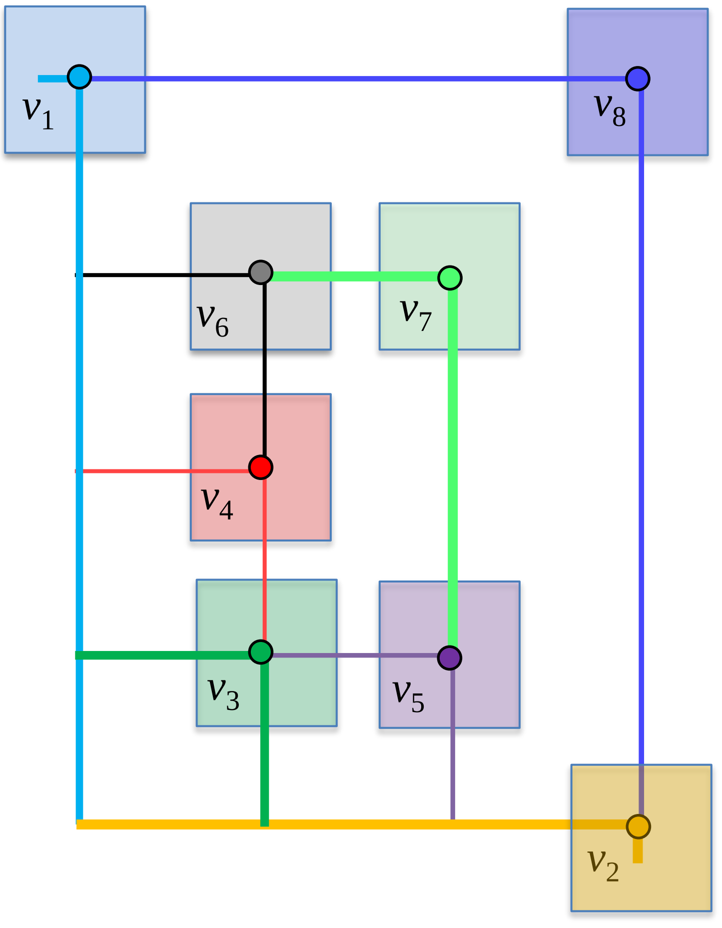
Phase 2 (Expansion)



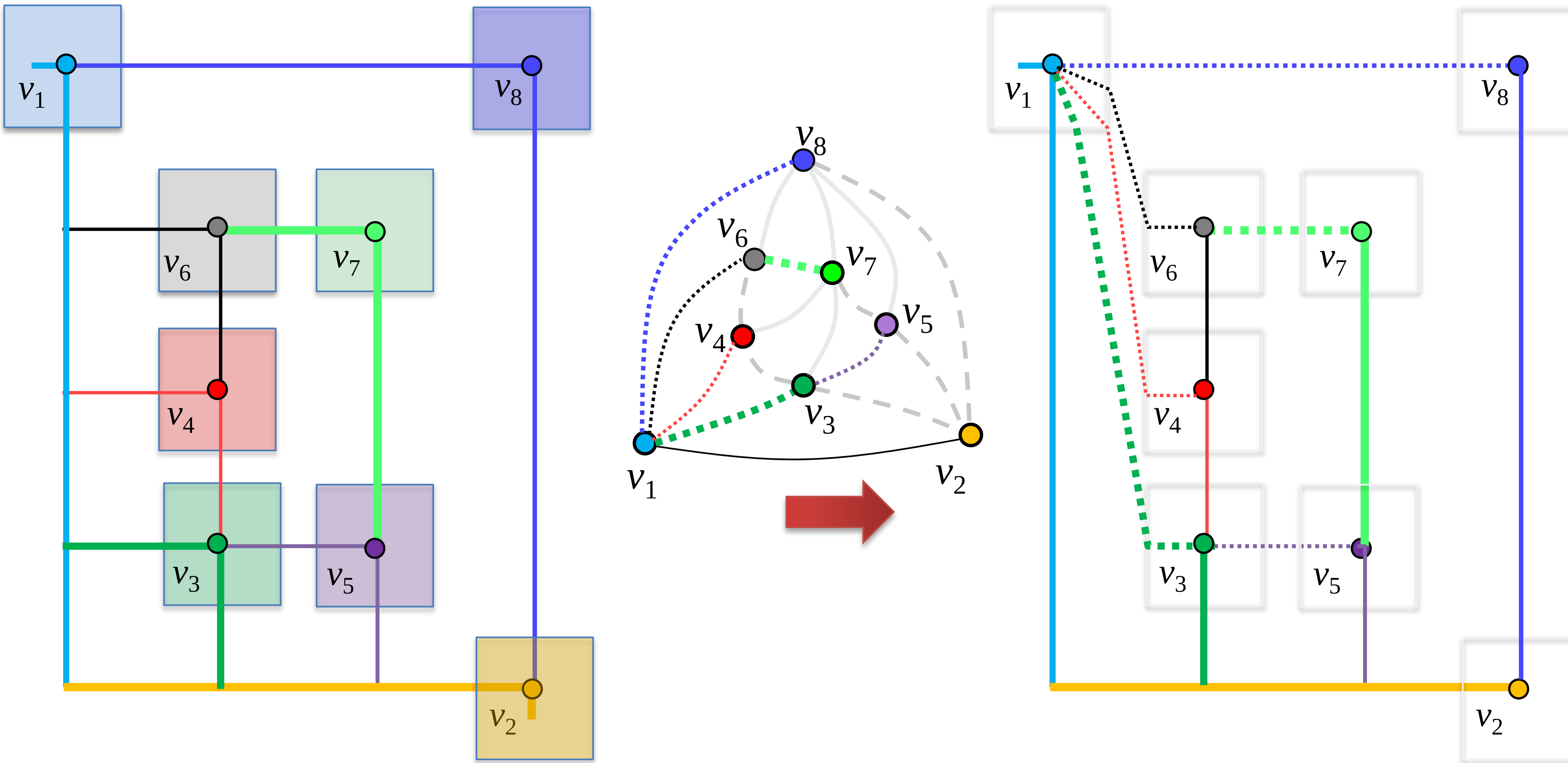
For each row l,

- let r be the vertex on l with maximum degree.
- insert floor(d_r/2) rows to each side of l.

Phase 2 (Expansion)

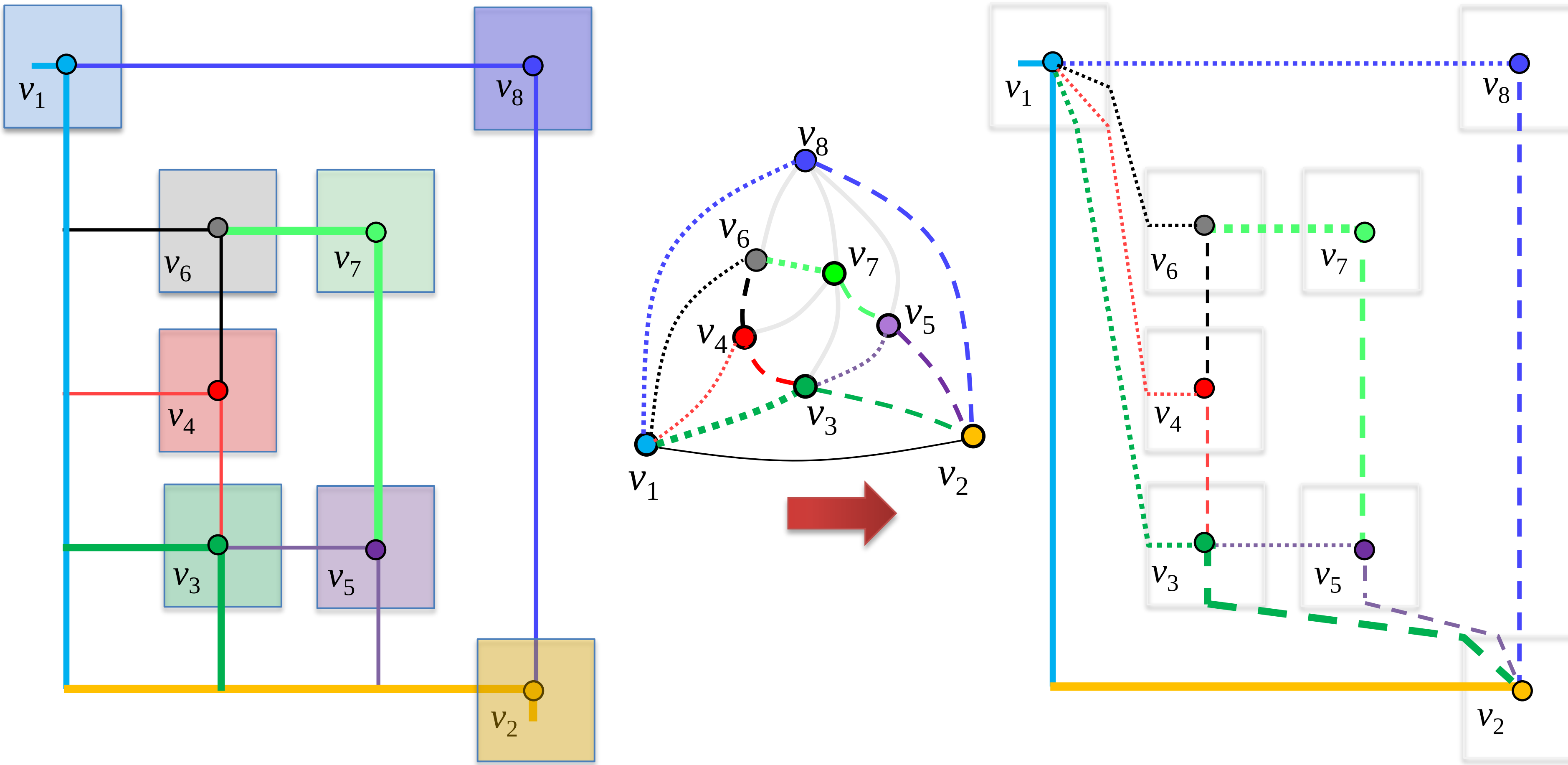


Phase 3 (Edge Routing)



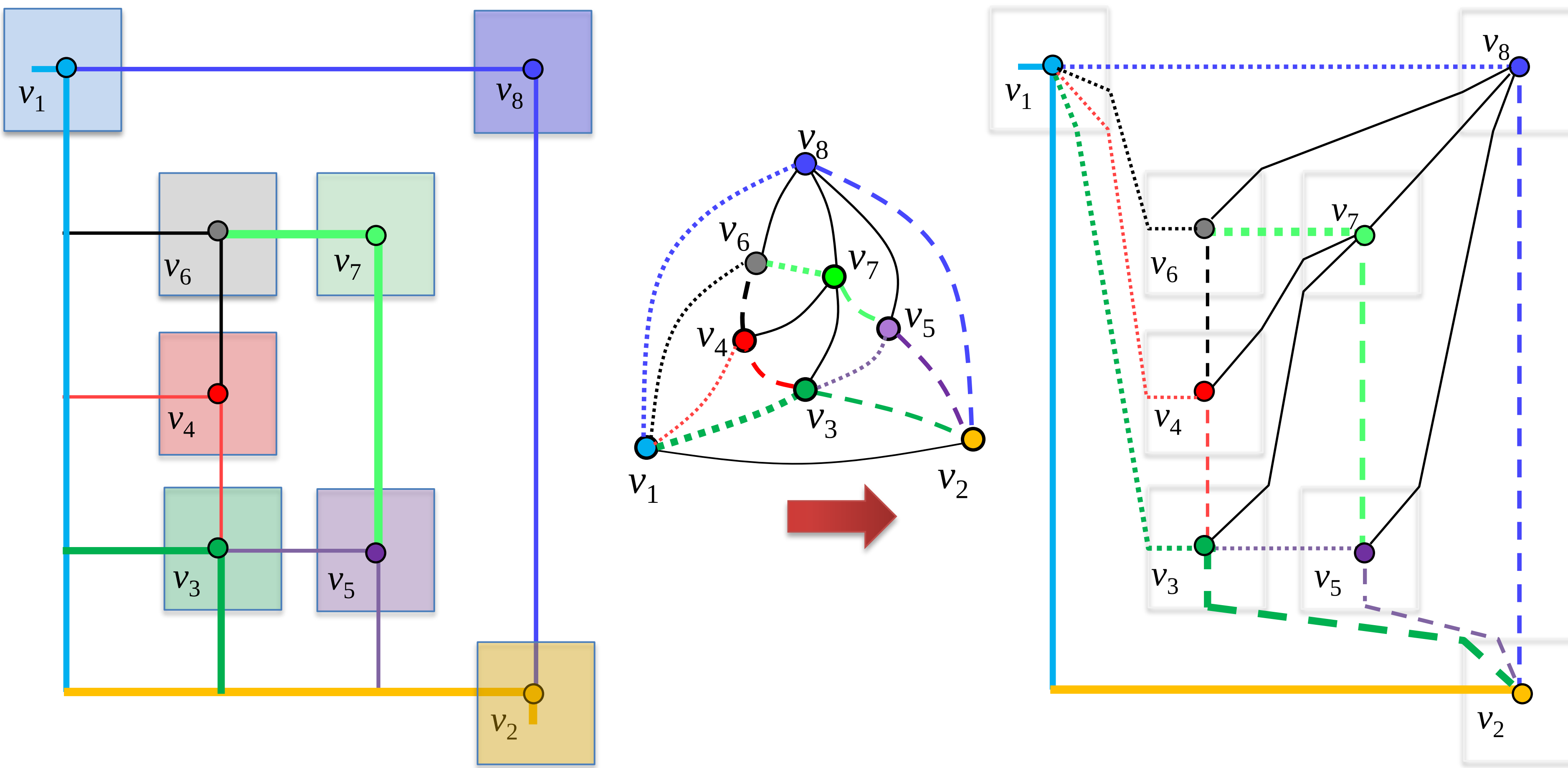
Drawing T_{left}

Phase 3 (Edge Routing)



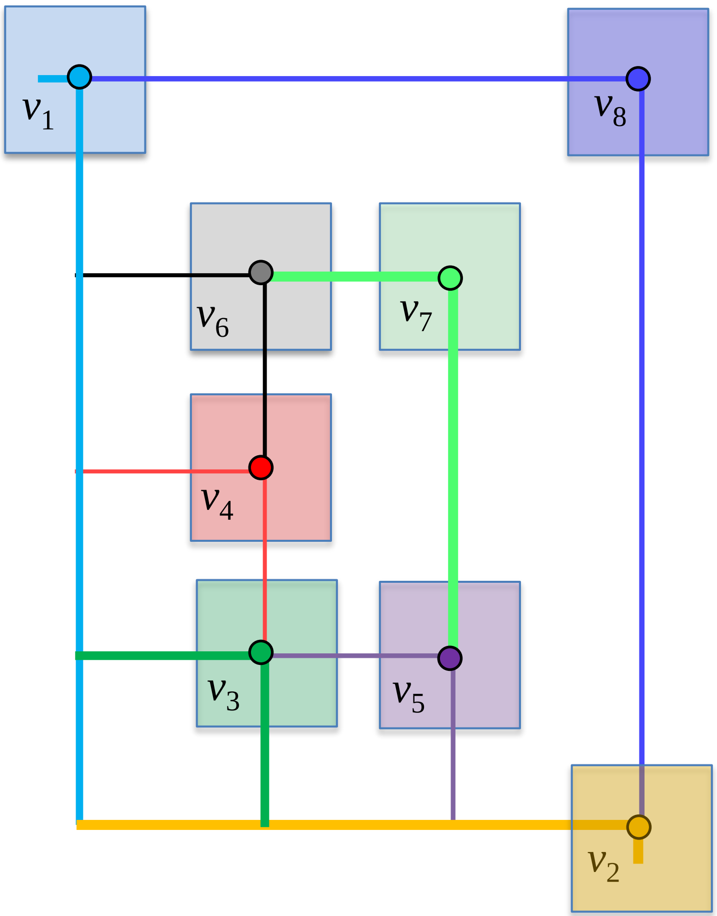
Drawing T_{right}

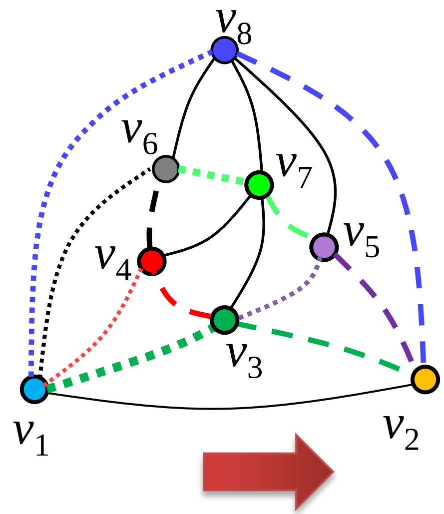
Phase 3 (Edge Routing)

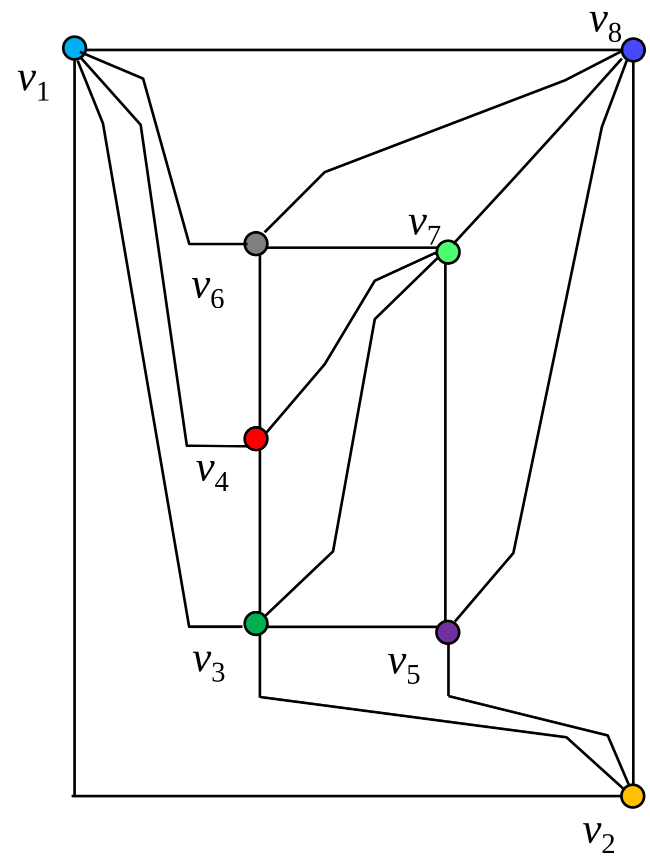


Drawing T_{mid}

Phase 3 (Edge Routing)

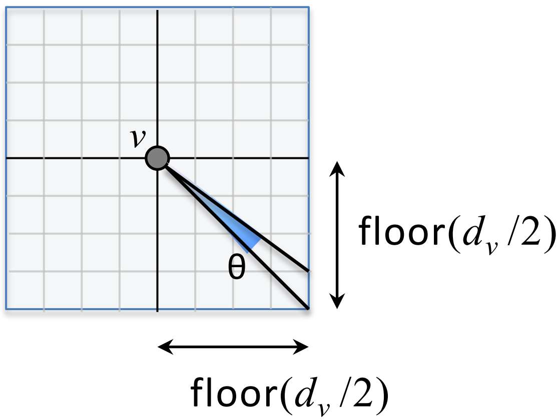


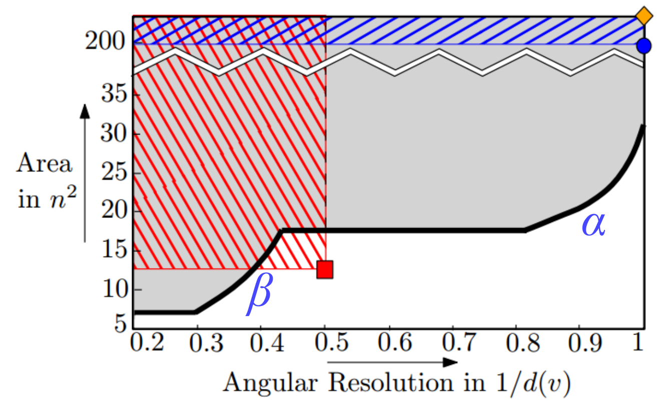




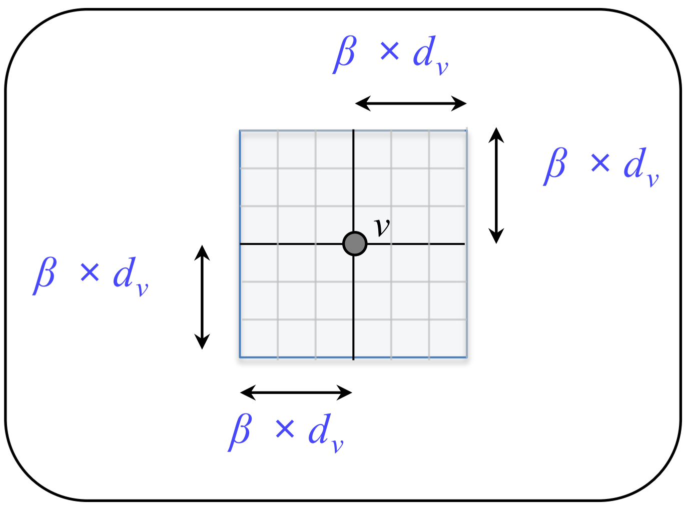
Final Output

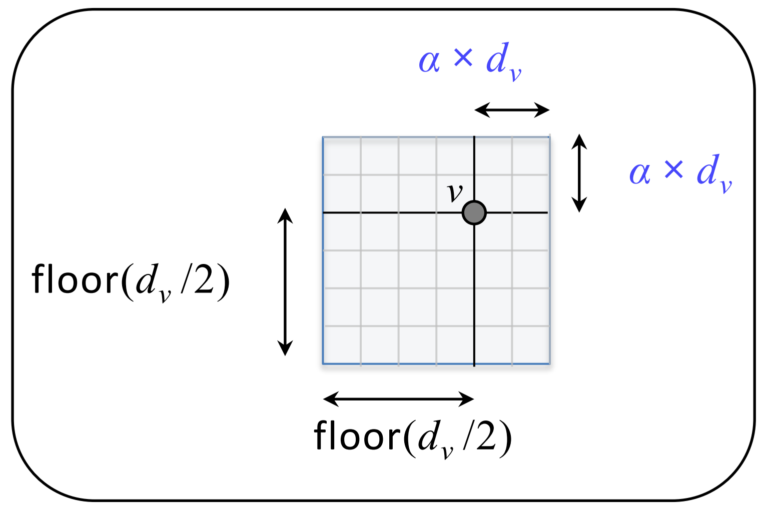
Where does the trade-off come from?





… Because we have other options ☺





Area	**Resolution**
$(6\beta + 2/3)^2 n^2$	$\frac{\beta}{d_v(\beta^2+1)}$

Area	**Resolution**
$(6\alpha+8/3)^2 n^2$	$\frac{\alpha}{d_v(\alpha^2+1/4)}$

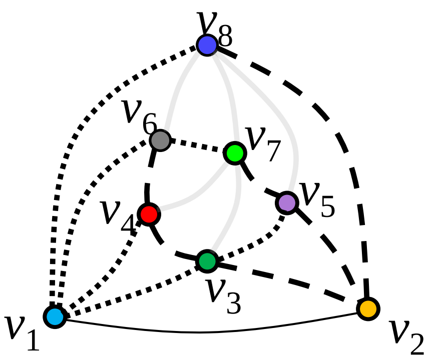
Future Research

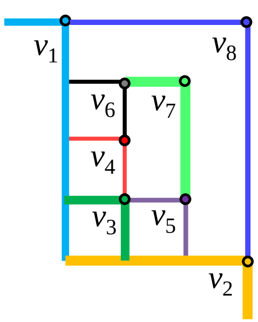
Better Trade-offs: Can we achieve better trade-offs between area and angular resolution for 2-bend polyline drawings? Find such trade-offs for 1-bend and 0-bend drawings.

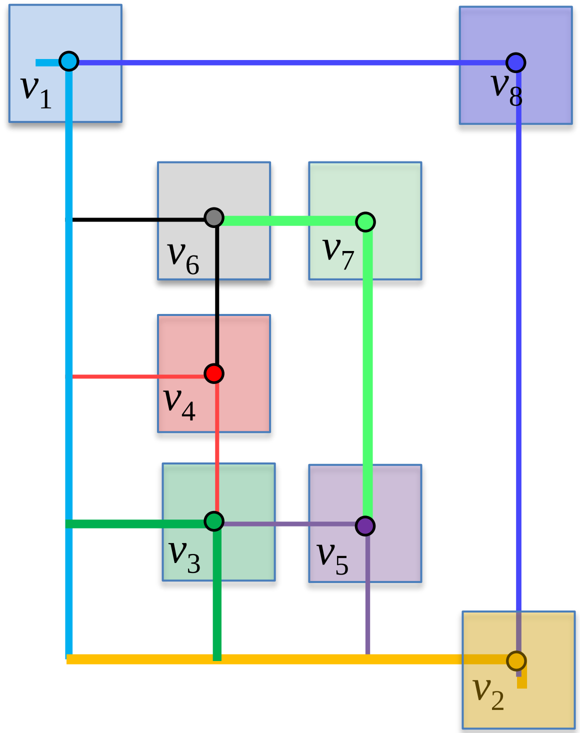
Generalization: Examine trade-offs between area and angular resolution for more general graphs such as 1-planar graphs and thickness 2-graphs.

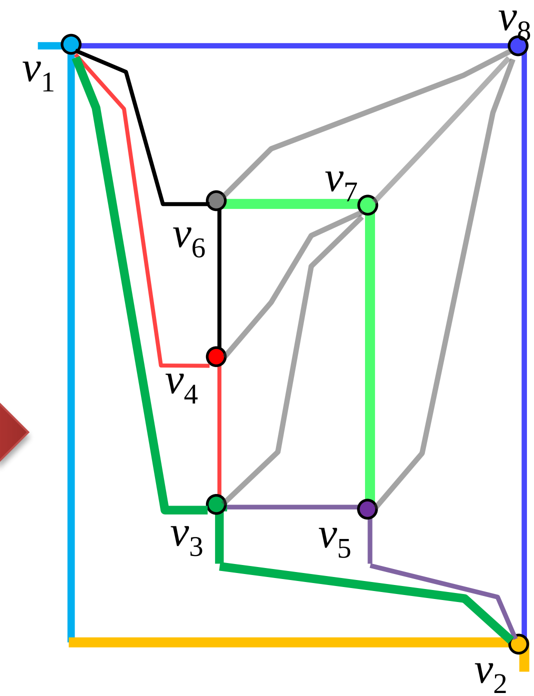
Different Aesthetics: Examine smooth trade-offs among different aesthetic criteria for other styles of drawing graphs?

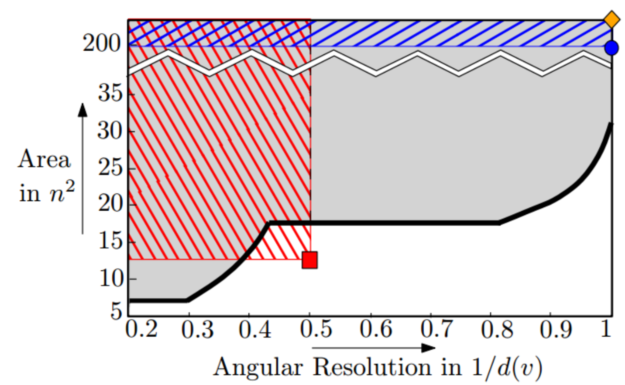
Thank You..











Better Trade-offs?

Generalization?

Different Aesthetics and Styles?