Infectious Disease Models 3

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Key Quantities for Infectious Disease Models: Parameters

- Contacts per susceptible per unit time: c
  - e.g. 20 contacts per month
  - This is the number of contacts a given susceptible will have with *anyone*

- Per-infective-with-susceptible-contact transmission probability: β
  - This is the per-contact likelihood that the pathogen will be transmitted from an infective to a susceptible with whom they come into a single contact.

Recall: Our model

- Set
  - c=10 (people/month)
  - β=0.04 (4% chance of transmission per S-I contact)
  - μ=10
  - Birth and death rate=0
  - Initial infectives=1, other 1000 susceptible

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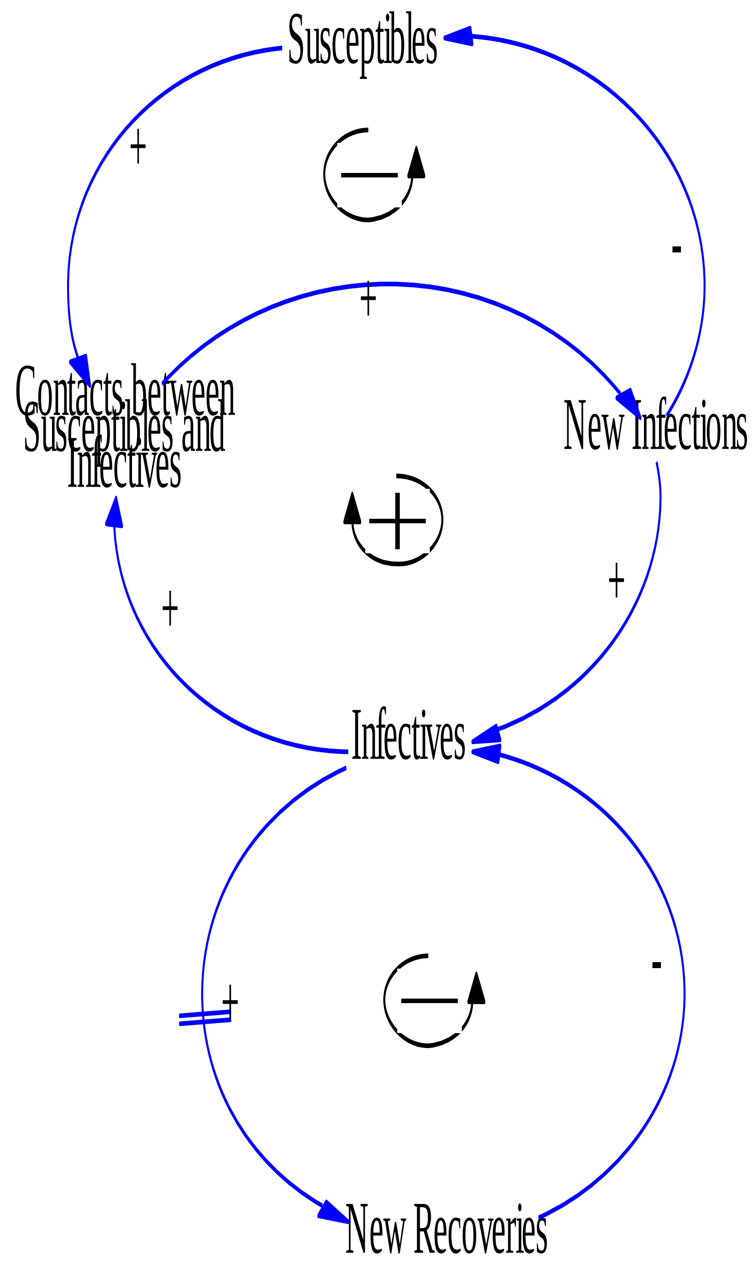
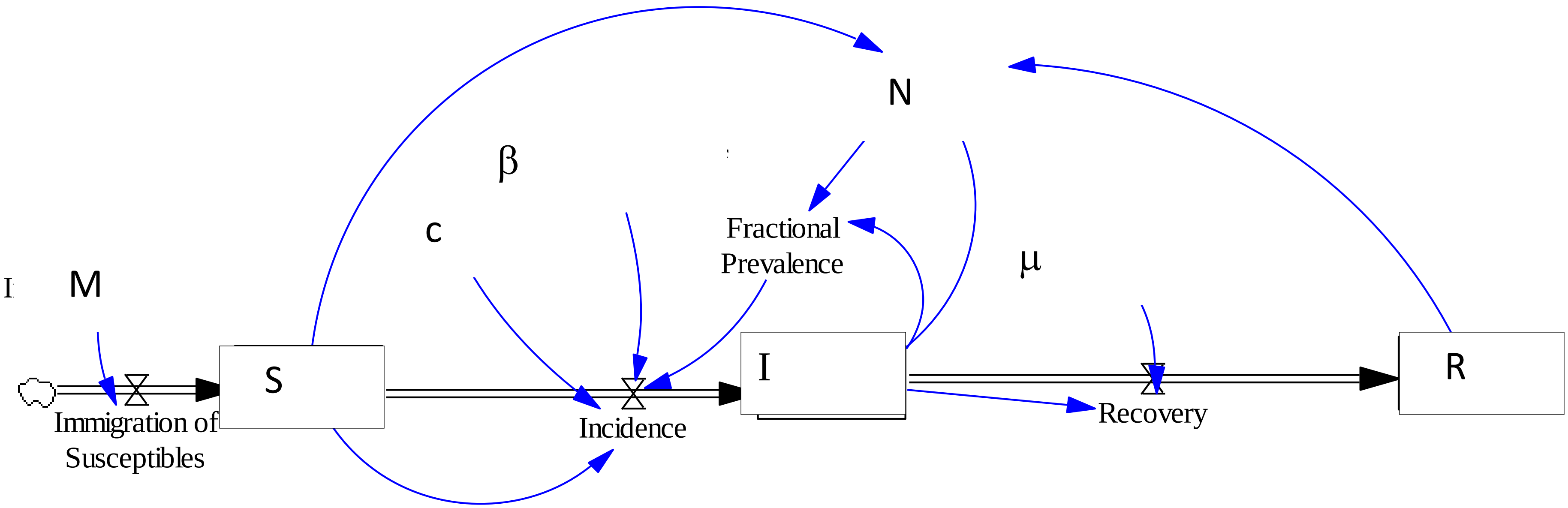
Let’s adjust several things

- Transmissibility (originally 0.04)
  - To 0.10
  - To 0.20
- Duration of infection (originally 10)
  - To 20
  - To 30

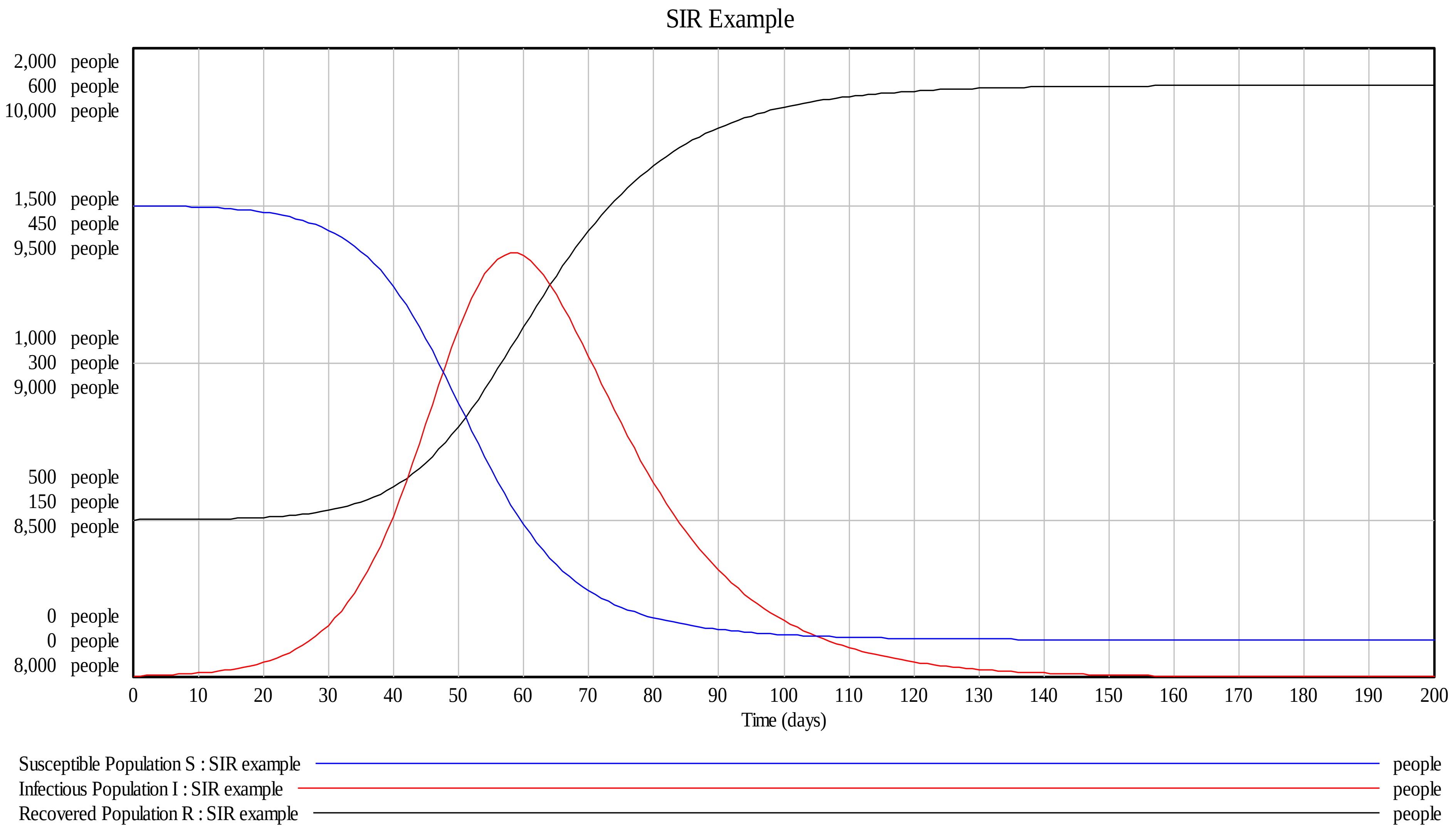
Associated Feedbacks



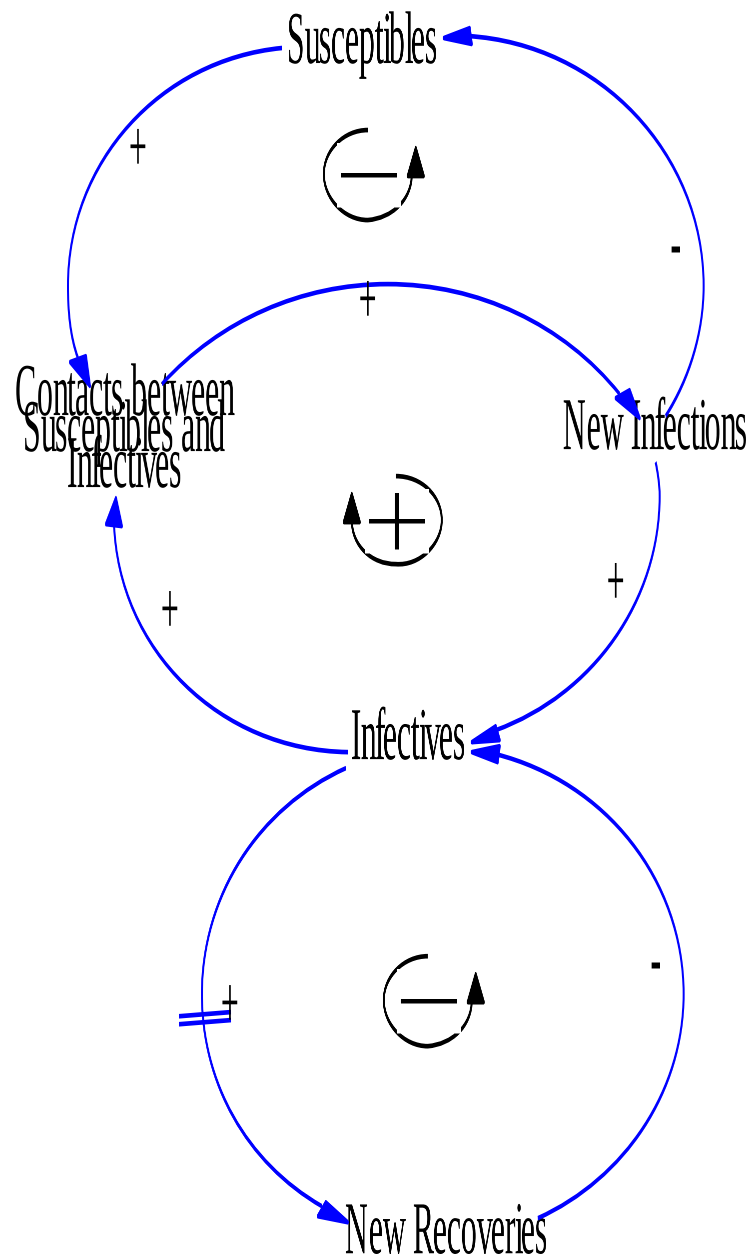
Mathematical Notation



Example Dynamics of SIR Model (No Births or Deaths)



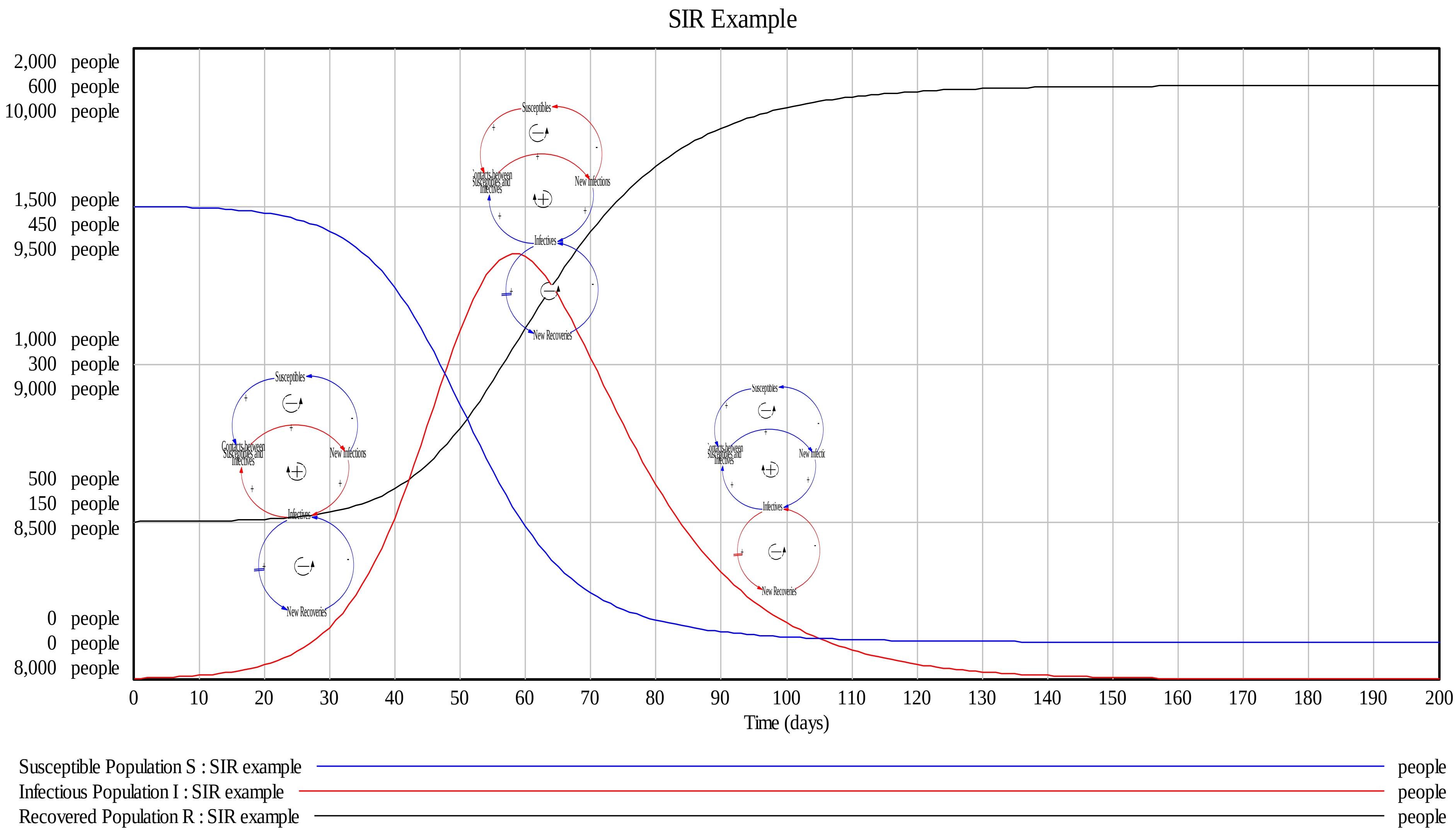
Shifting Feedback Dominance



Explaining the Stock & Flow Dynamics: Infectives&Susceptibles

- Initially
  - Each infective infects $c(S/N)\beta \approx c\beta$ people on average for each time unit – the maximum possible rate
  - The rate of recoveries is 0
- In short term
  - # Infectives grows (quickly)=> rate of infection rises quickly
    - (Positive feedback!)
  - Susceptibles starts to decline, but still high enough that each infective is surrounded overwhelmingly by susceptibles, so efficient at transmitting
- Over time, more infectives, and fewer Susceptibles
  - Fewer S around each I =>Rate of infections per I declines
  - Many infectives start recovering => slower rise to I
- "Tipping point": # of infectives plateaus
  - Rate of infections = Rate of recoveries
  - Each infective infects exactly one "replacement" before recovering
- In longer term, declining # of infectives&susceptibles=> Lower & lower rate of new infections (negative feedback!)
- Change in I dominated by recoveries => goal seeking to 0 (negative feedback!)

Shifting Feedback Dominance



Damped Oscillatory Behavior

- Modify model to have births and deaths, with an annual birth-and-death rate
- Set Model/Settings/Final Time to 1000 (long time frame)
- In “Synthesim” (“Running man”) mode, set Birth/death rates
  - 0.02
  - 0.05
  - 0.07
  - 0.09

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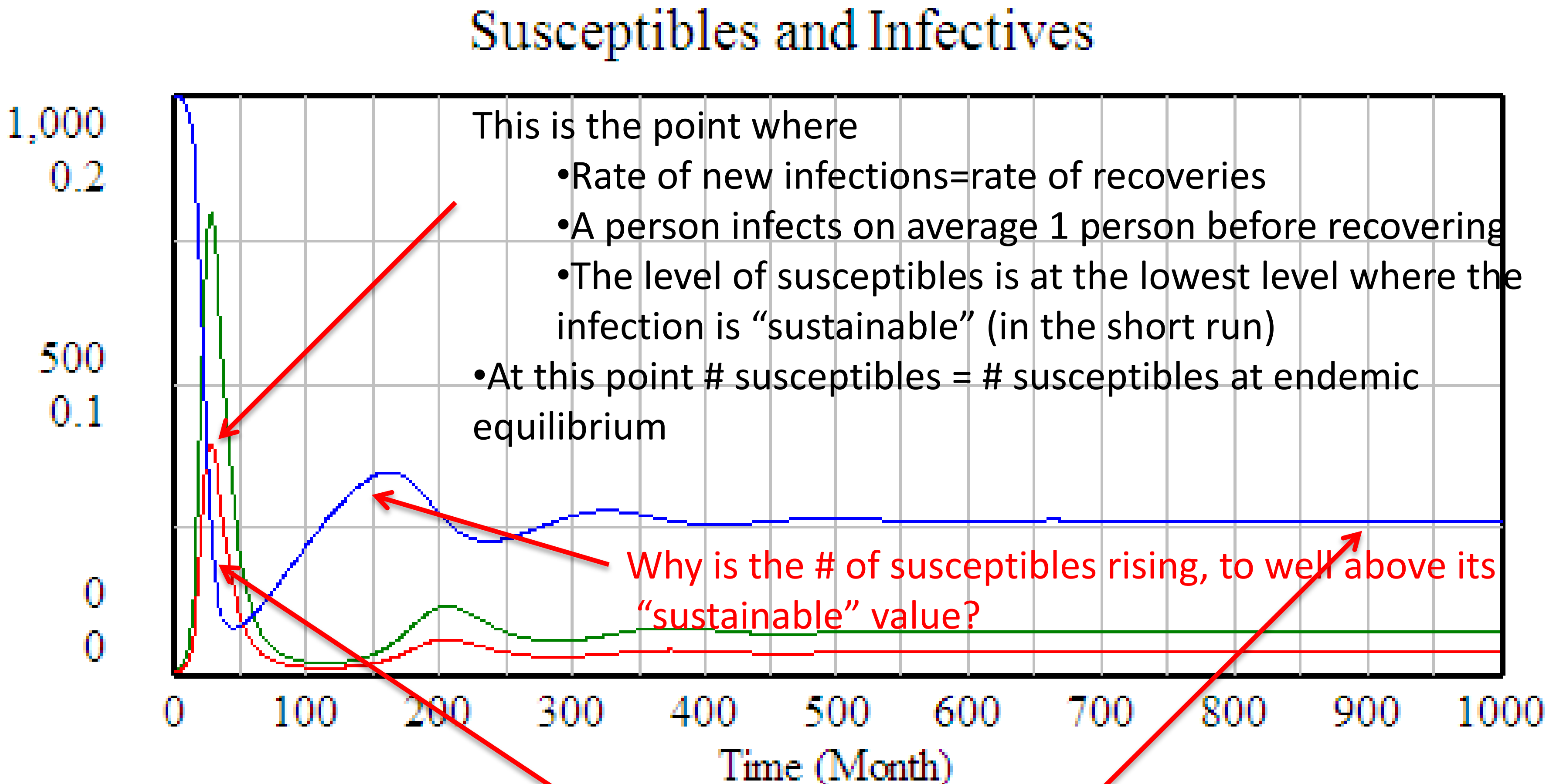
Delays

- For a while after infectives start declining (i.e. susceptibles are below sustainable endemic value), they still deplete susceptibles sufficiently for susceptibles to decline
- For a while after susceptibles are rising (until susceptibles=endemic value), infectives will still decline
- For a while after infectives start rising, births > # of infections =>susceptibles will rise to a peak well above endemic level



This is the point where

- Rate of new infections=rate of recoveries
- A person infects on average 1 person before recovering
- The level of susceptibles is at the lowest level where the infection is "sustainable" (in the short run)

- At this point # susceptibles = # susceptibles at endemic equilibrium

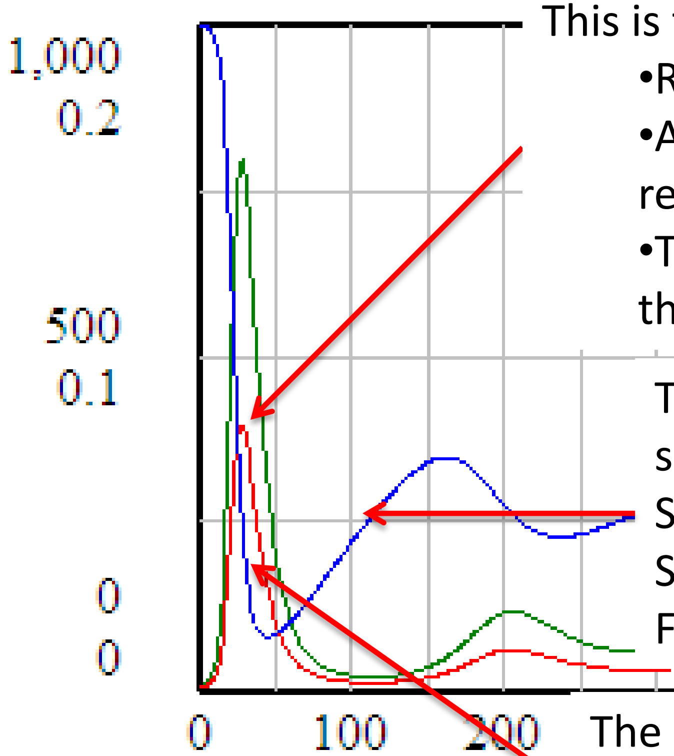
Why is the # of susceptibles rising, to well above its "sustainable" value?

Why is the # of susceptibles still declining?

This fraction of susceptibles at endemic equilibrium is the minimum "sustainable" value of susceptible – i.e. the value where the properties above hold.

- Above this fraction of susceptibles, the # infected will rise
- Below this fraction of susceptibles, the # infected will fall

Susceptibles and Infectives



This is the point where

- Rate of new infections=rate of recoveries
- A person infects on average 1 person before recovering
- The level of susceptibles is at the lowest level where the infection is “sustainable” (in the short run)

The rise is occurring because infectives are so low that so few infections occur that births >infections+deaths. S rises above the sustainable value because infectives are Still in decline until that point – so infectives remain low For a while!

The susceptibles are still declining here because the large # of infectives still causes enough infections that
rate of immigration < rate of infections + deaths

Equilibrium Behaviour

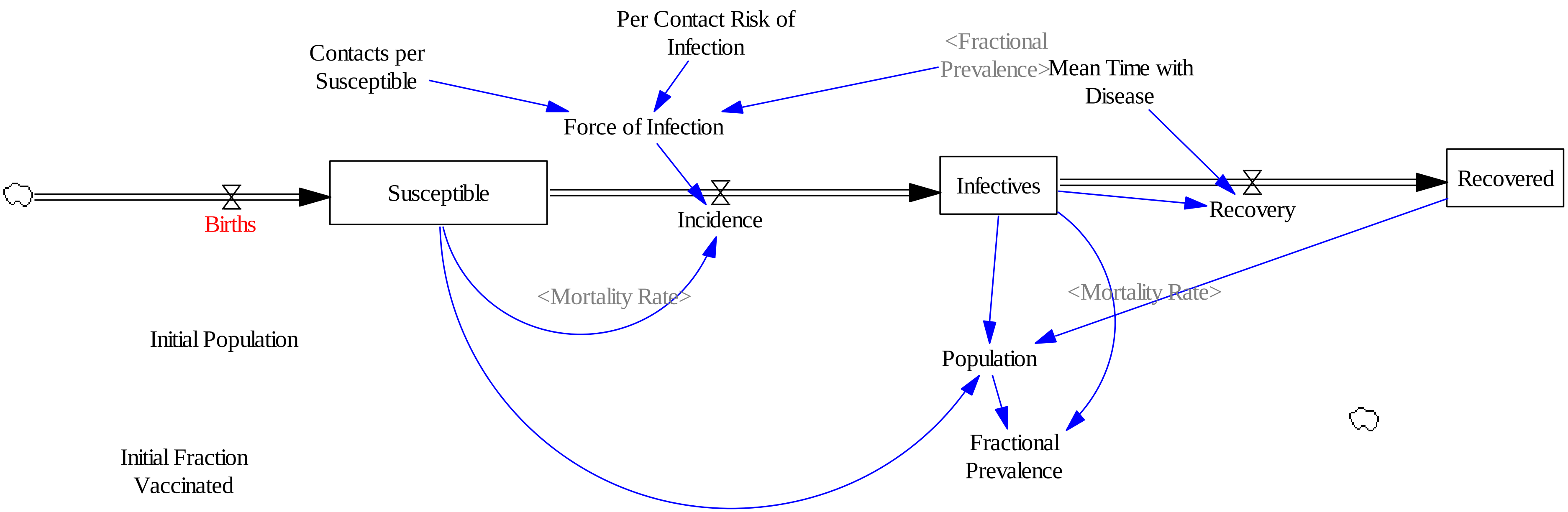
- With Births & Deaths, the system can approach an “endemic equilibrium” where the infection stays circulating in the population – but in balance
- The balance is such that (simultaneously)
  - The rate of new infections = The rate of immigration
    - Otherwise # of susceptibles would be changing!
  - The rate of new infections = the rate of recovery
    - Otherwise # of infectives would be changing!

Tipping Point

- Now try setting transmission rate β to 0.005

Recall: Kendrick-McKermack Model

- Partitioning the population into 3 broad categories:
  - Susceptible (S)
  - Infectious (I)
  - Removed (R)


Per Contact Risk of Infection
Contacts per Susceptible
<Fractional Prevalence>
Mean Time with Disease
Force of Infection
Susceptible
Infectives
Recovered
Births
Incidence
Recovery
<Mortality Rate>
<Mortality Rate>
Initial Population
Population
Fractional Prevalence
Initial Fraction Vaccinated

Shorthand for Key Quantities for Infectious Disease Models: Stocks

- *I* (or *Y*): Total number of infectives in population
  - This could be just one stock, or the sum of many stocks in the model (e.g. the sum of separate stocks for asymptomatic infectives and symptomatic infectives)
- N: Total size of population
  - This will typically be the sum of all the stocks of people
- S (or *X*): Number of susceptible individuals

Intuition Behind Common Terms

- I/N: The Fraction of population members (or, by assumption, contacts!) that are infective
  - Important: Simplest models assume that this is also the fraction of a given susceptible’s contacts that are infective! Many sophisticated models relax this assumption
- c(I/N): Number of *infectives* that come into contact with a susceptible in a given unit time
- c(I/N)β: “Force of infection”: *Likelihood a given susceptible will be infected per unit time*
  - The idea is that if a given susceptible comes into contact with c(I/N) infectives per unit time, and if each such contact gives β likelihood of transmission of infection, then that susceptible has roughly a total likelihood of c(I/N) β of getting infected per unit time (e.g. month)

Key Term: Flow Rate of New Infections

- This is the key form of the equation in many infectious disease models
- Total # of susceptiblesinfected per unit time

  # of Susceptibles * "Likelihood" a given susceptible will be infected per unit time = S*("*Force of Infection*") =S(c(I/N)β)

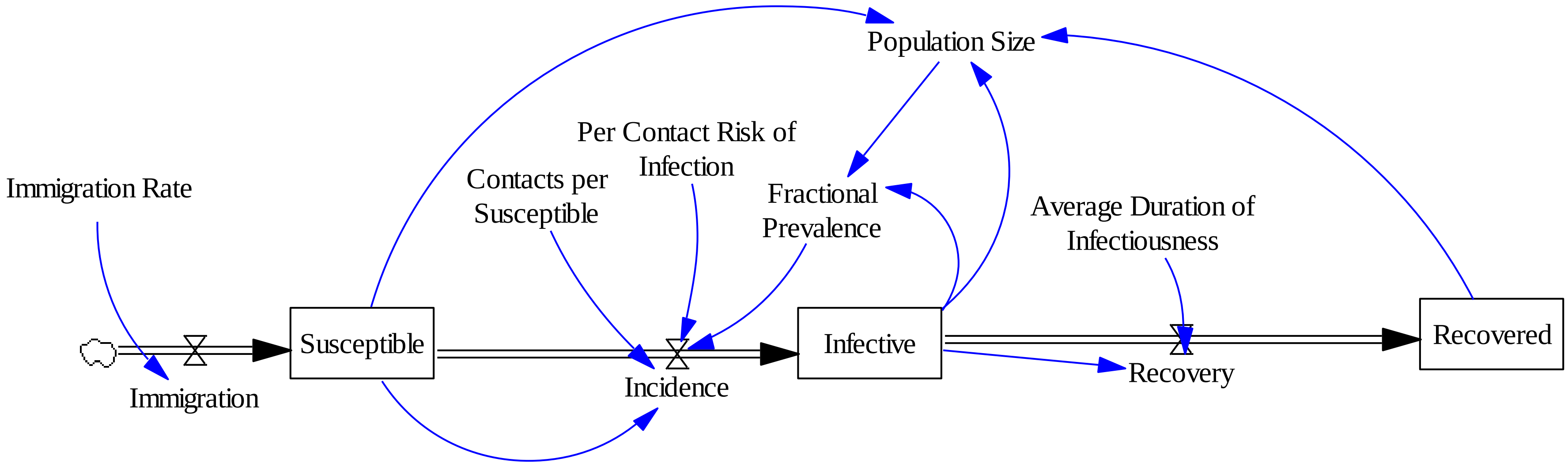
  – Note that this is a term that multiplies both S and I !
    - This is much different than the purely linear terms on which we have previously focused

  – "Likelihood" is actually a likelihood density (e.g. can be >1 – indicating that mean time to infection is <1)

Another Useful View of this Flow

- Recall: Total # of susceptibles infected per unit time = # of Susceptibles * "Likelihood" a given susceptible will be infected per unit time = S*("*Force of Infection*") = $S(c(I/N)\beta)$
- The above can also be phrased as the following: $S(c(I/N)\beta)=I(c(S/N)\beta)$=# of Infectives * Average # susceptibles infected per unit time by each infective
- This implies that as # of susceptibles falls=>># of susceptibles surrounding each infective falls=>>the rate of new infections falls ("Less fuel for the fire" leads to a smaller burning rate

Basic Model Structure



Underlying Equations

$$\dot{S} = M - c\left(\frac{I}{N}\right)\beta S$$

$$\dot{I} = c\left(\frac{I}{N}\right)\beta S - \frac{I}{\mu}$$

$$\dot{R} = \frac{I}{\mu}$$

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