Infectious Disease Models 4: Basic Quantities of Mathematical Infectious Disease Epidemiology

Nathaniel Osgood

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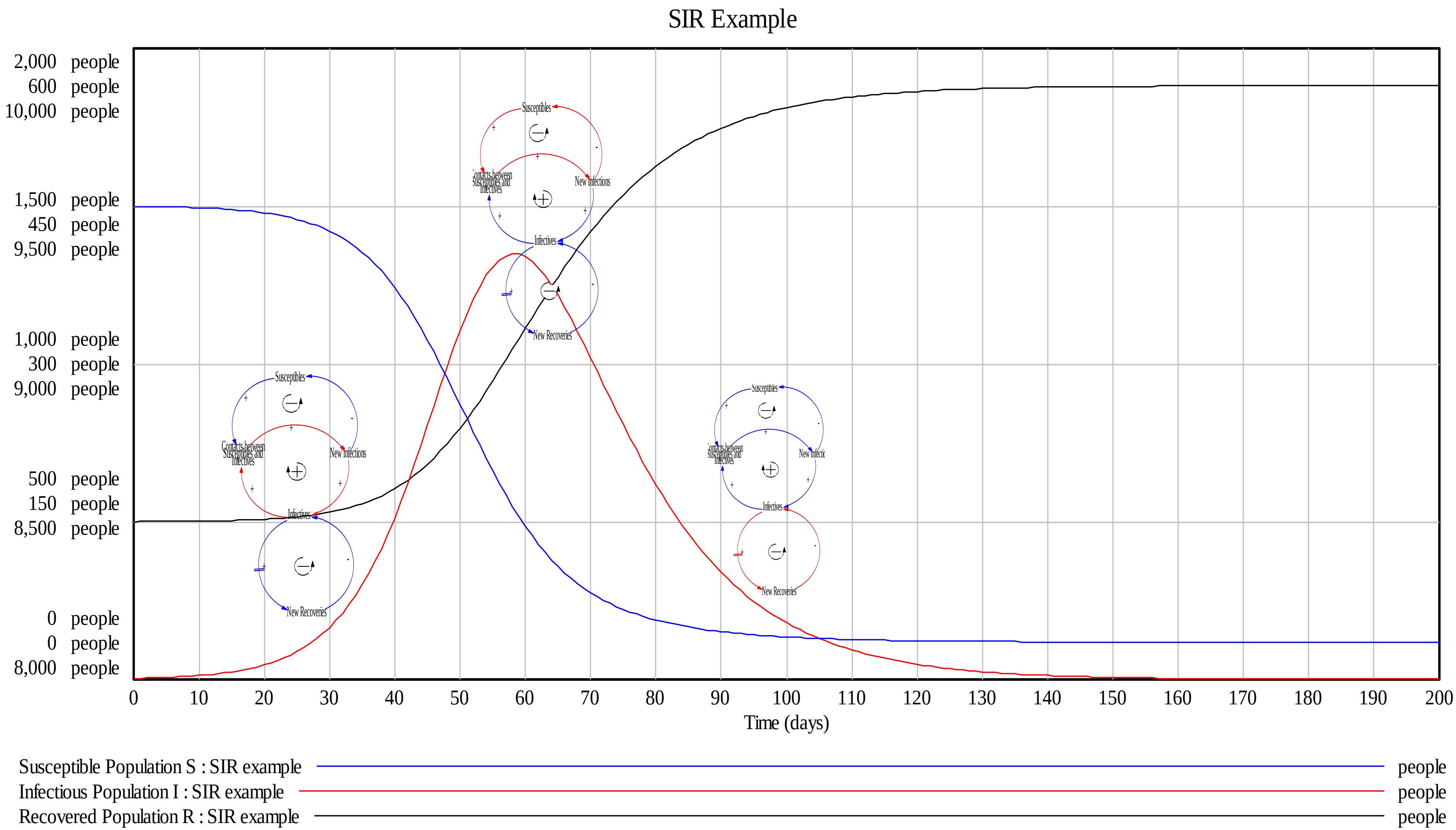
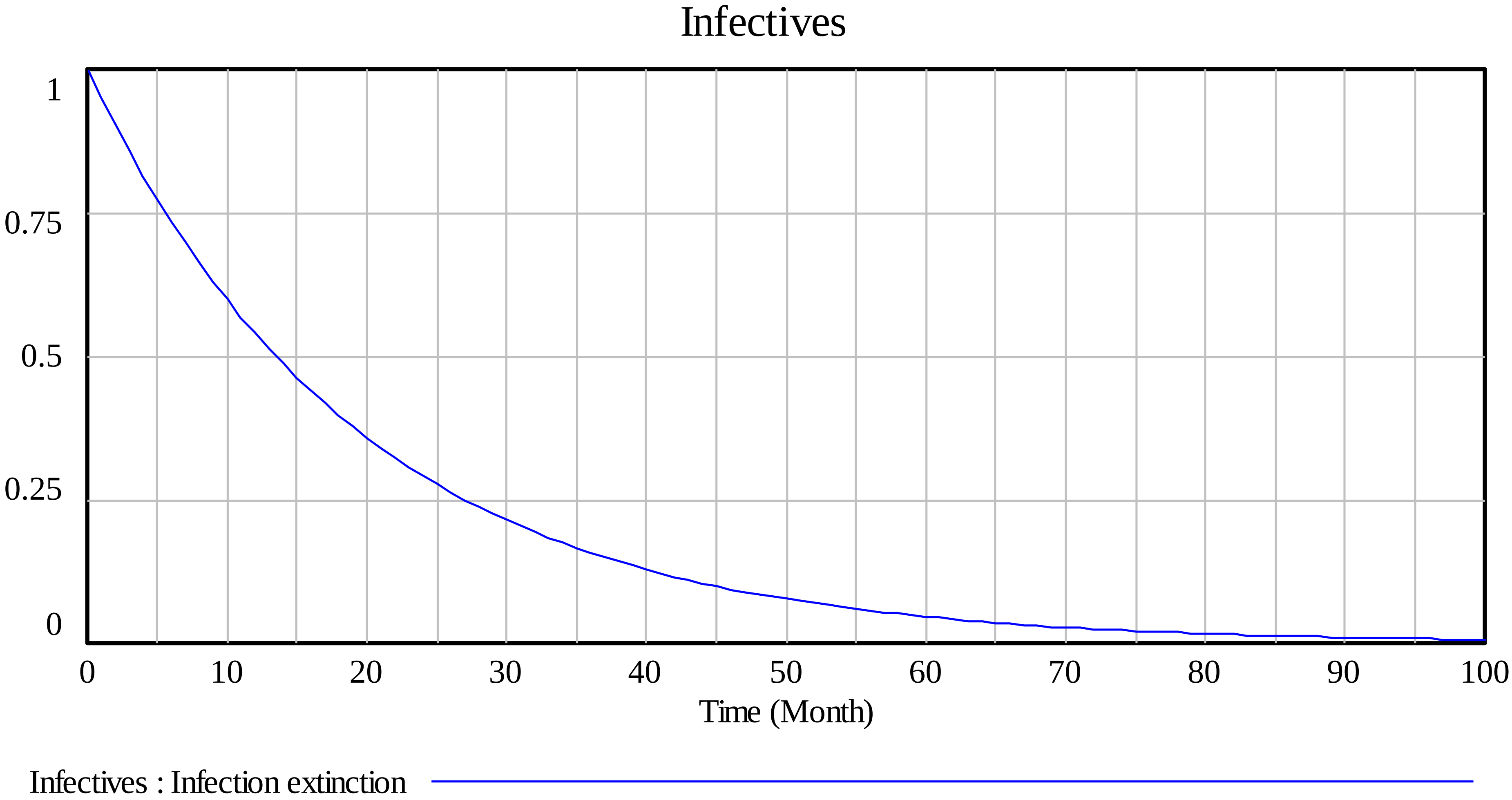
Recall: Closed Population (No Birth & Death)

- Infection always dies out in the population
- Some infections will take longer to die out
- There is a "tipping point" between two cases
  - # of people infected declines out immediately
  - Infection causes an outbreak before the infection dies down (# of people infected rises and then falls)

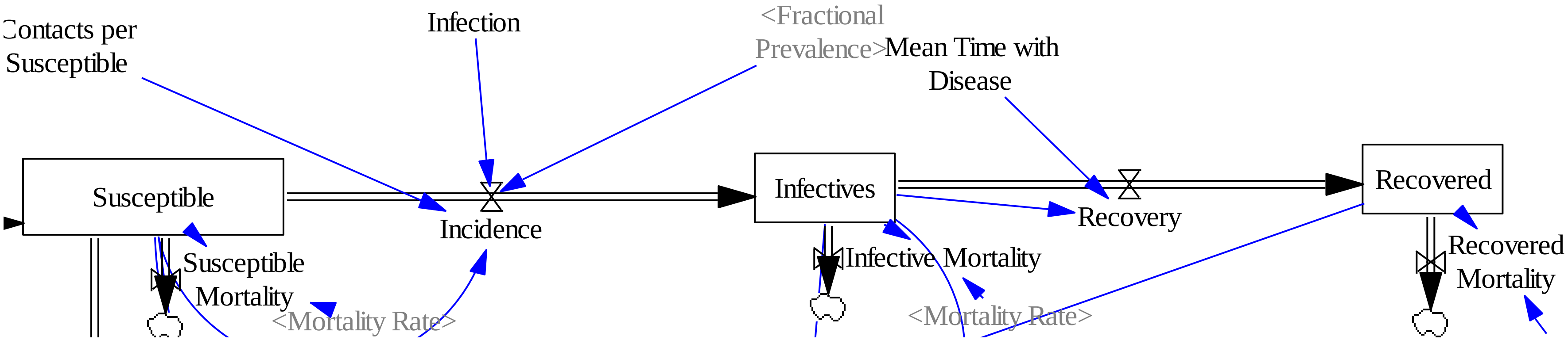
Case 1: Outbreak



Case 2: Infection declines immediately



Recall: Simple Model Incorporating Population Turnover



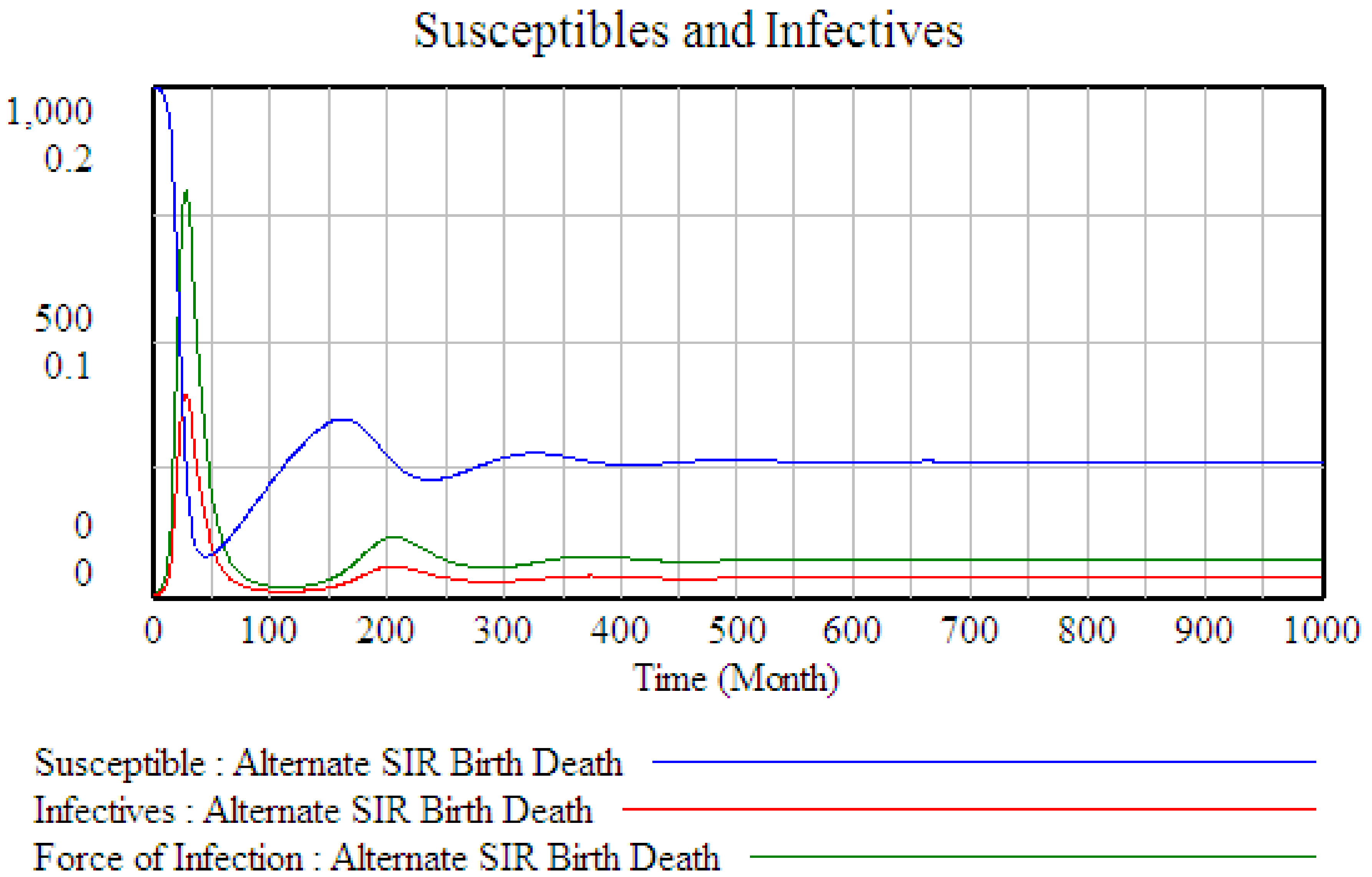
Recall: Our model

- Set
  - c=10 (people/month)
  - β=0.04 (4% chance of transmission per S-I contact)
  - μ=10
  - Birth and death rate= 0
  - Initial infectives=1, other 1000 susceptible

Recall: Our model

- Set
  - c=10 (people/month)
  - β=0.04 (4% chance of transmission per S-I contact)
  - μ=10
  - Birth and death rate=0.02
  - Initial infectives=1, other 1000 susceptible

Here, the Infection Can Remain (Endemic)



Damped Oscillatory Behavior

- Modify model to have births and deaths, with an annual birth-and-death rate
- Set Model/Settings/Final Time to 1000 (long time frame)
- In “Synthesim” (“Running man”) mode, set Birth/death rates
  - 0.02
  - 0.05
  - 0.07
  - 0.01
  - 0.001

Equilibrium Behaviour

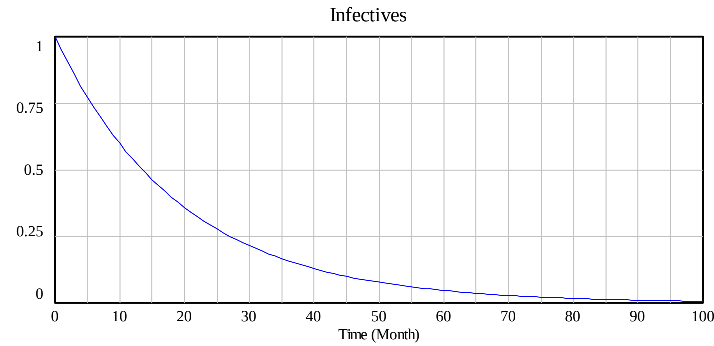
- With Births & Deaths, the system can approach an "endemic equilibrium" where the infection stays circulating in the population – but in balance
- The balance is such that (simultaneously)
  - The rate of new infections = The rate of immigration
    - Otherwise # of susceptibles would be changing!
  - The rate of new infections = the rate of recovery
    - Otherwise # of infectives would be changing!

Exploring the Tipping Point

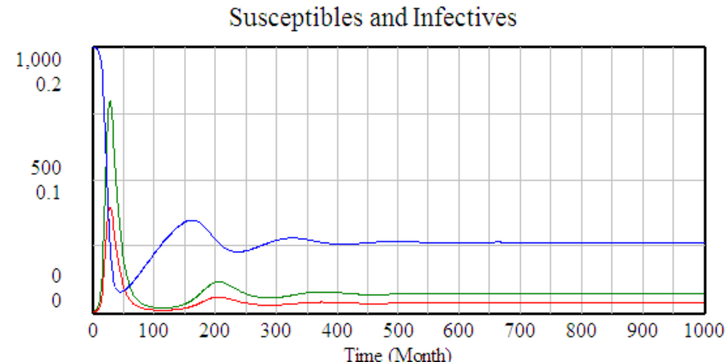
- Now try setting transmission rate β to 0.005

Infection Extinction

- As for the case with a closed population, an open population has two cases
  - Infection dies out immediately



  - Outbreak: Infection takes off
    - Here – in contrast to the case for a closed population – the infection will typically go to an endemic equilibrium



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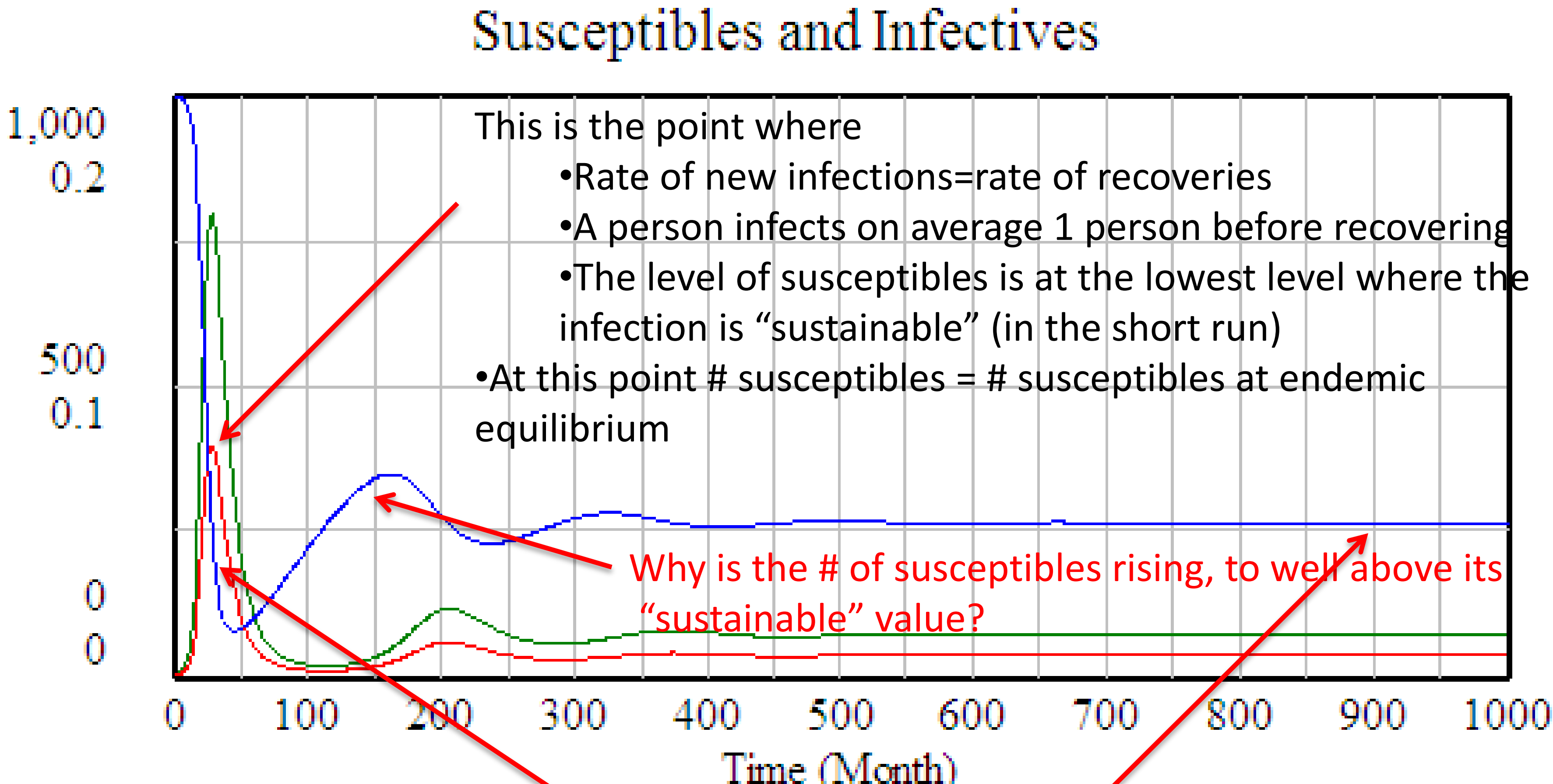
Delays

- For a while after infectives start declining (i.e. susceptibles are below sustainable endemic value), they still deplete susceptibles sufficiently for susceptibles to decline
- For a while after susceptibles are rising (until susceptibles=endemic value), infectives will still decline
- For a while after infectives start rising, births > # of infections =>susceptibles will rise to a peak well above endemic level

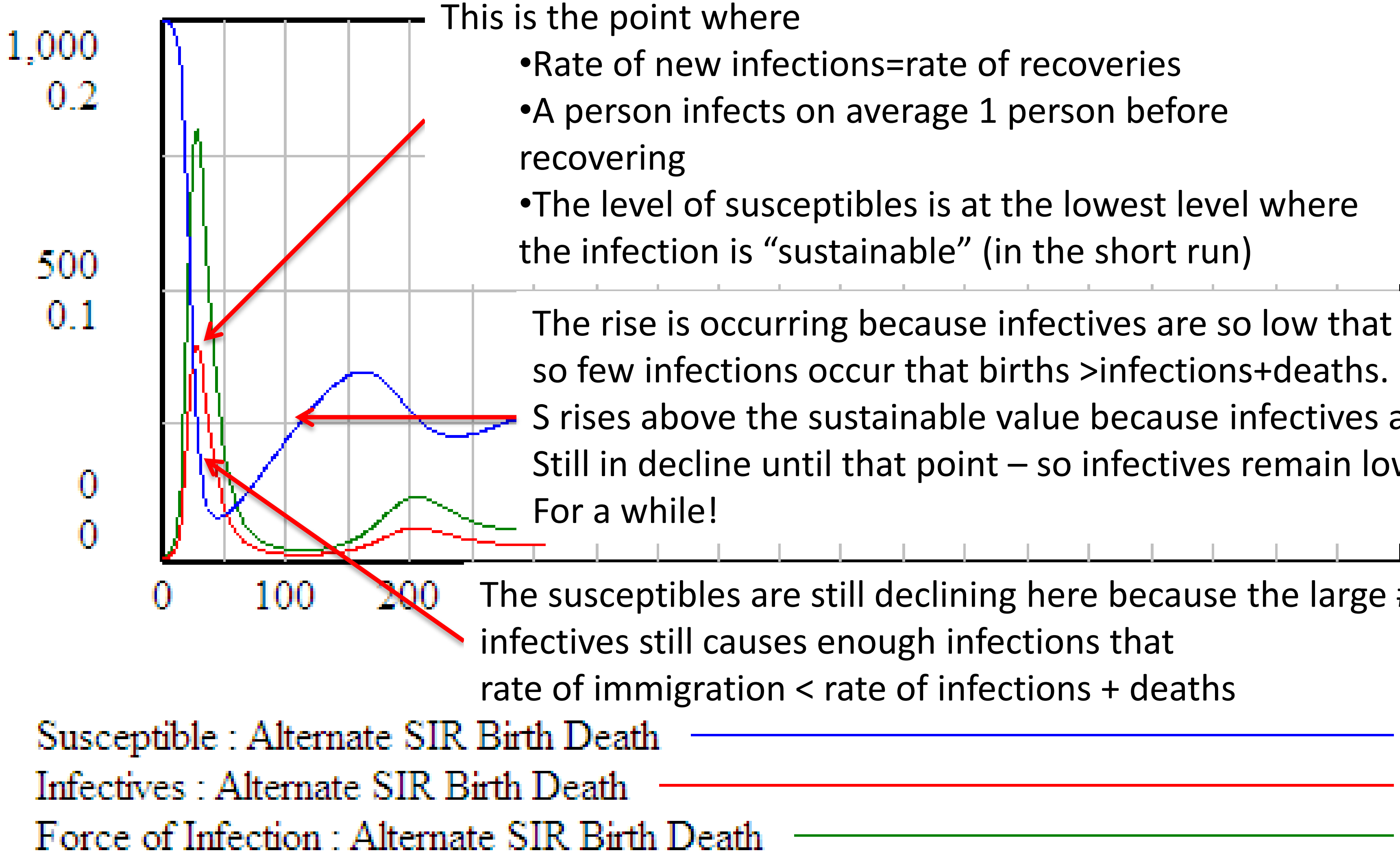


Why is the # of susceptibles still declining?

This fraction of susceptibles at endemic equilibrium is the minimum “sustainable” value of susceptible – i.e. the value where the properties above hold.

- Above this fraction of susceptibles, the # infected will rise
- Below this fraction of susceptibles, the # infected will fall

Susceptibles and Infectives



This is the point where

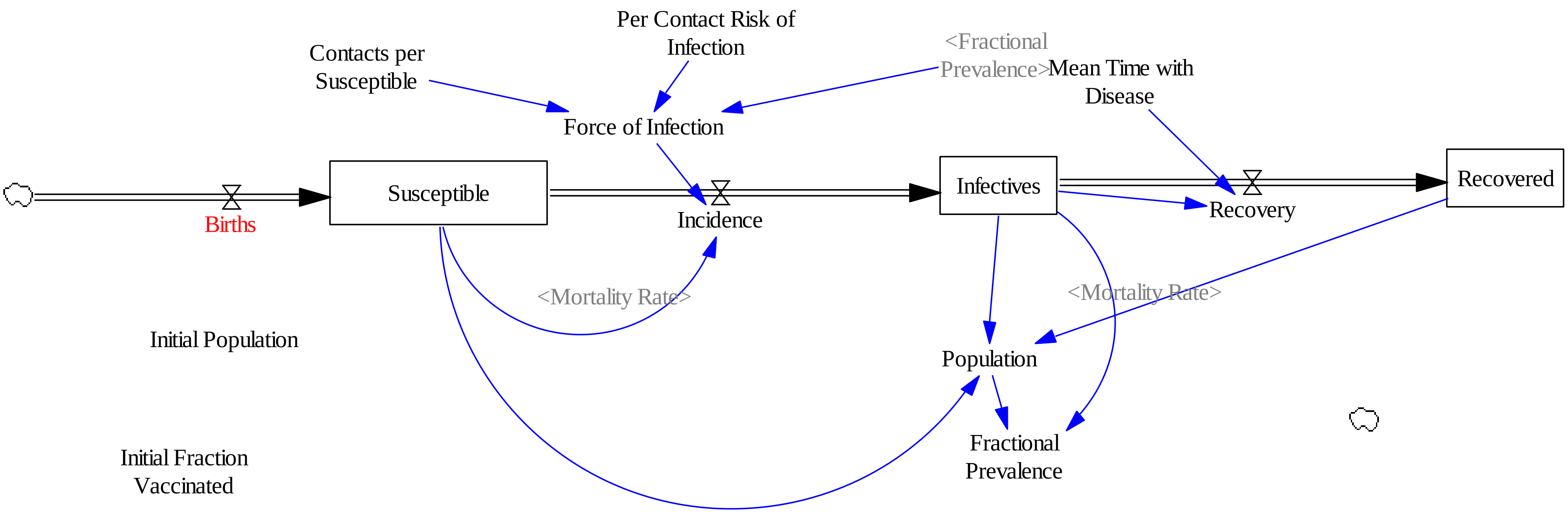
- Rate of new infections=rate of recoveries
- A person infects on average 1 person before recovering
- The level of susceptibles is at the lowest level where the infection is “sustainable” (in the short run)

The rise is occurring because infectives are so low that so few infections occur that births >infections+deaths. S rises above the sustainable value because infectives are Still in decline until that point – so infectives remain low For a while!

The susceptibles are still declining here because the large # of infectives still causes enough infections that rate of immigration < rate of infections + deaths

Recall: Kendrick-McKermack Model

- Partitioning the population into 3 broad categories:
  - Susceptible (S)
  - Infectious (I)
  - Removed (R)


Per Contact Risk of Infection
Contacts per Susceptible
<Fractional Prevalence>
Mean Time with Disease
Force of Infection
Susceptible
Infectives
Recovered
Births
Incidence
Recovery
<Mortality Rate>
<Mortality Rate>
Initial Population
Population
Initial Fraction Vaccinated
Fractional Prevalence

Shorthand for Key Quantities for Infectious Disease Models: Stocks

- *I* (or *Y*): Total number of infectives in population
  - This could be just one stock, or the sum of many stocks in the model (e.g. the sum of separate stocks for asymptomatic infectives and symptomatic infectives)
- N: Total size of population
  - This will typically be the sum of all the stocks of people
- S (or *X*): Number of susceptible individuals

Intuition Behind Common Terms

- I/N: The Fraction of population members (or, by assumption, contacts!) that are infective
  - Important: Simplest models assume that this is also the fraction of a given susceptible's contacts that are infective! Many sophisticated models relax this assumption
- c(I/N): Number of *infectives* that come into contact with a susceptible in a given unit time
- c(I/N)β: "Force of infection": *Likelihood a given susceptible will be infected per unit time*
  - The idea is that if a given susceptible comes into contact with c(I/N) infectives per unit time, and if each such contact gives β likelihood of transmission of infection, then that susceptible has roughly a total likelihood of c(I/N) β of getting infected per unit time (e.g. month)

A Critical Throttle on Infection Spread: Fraction Susceptible (*f*)

- The fraction susceptible (here, S/N) is a key quantity limiting the spread of infection in a population
  - Recognizing its importance, we give this name *f* to the fraction of the population that issusceptible

Key Term: Flow Rate of New Infections

- This is the key form of the equation in many infectious disease models
- Total # of susceptiblesinfected per unit time

  # of Susceptibles * "Likelihood" a given susceptible will be infected per unit time = S*("*Force of Infection*") $=S(c(I/N)\beta)$

  – Note that this is a term that multiplies both S and I !
    - This is much different than the purely linear terms on which we have previously focused

  – "Likelihood" is actually a likelihood density (e.g. can be >1 – indicating that mean time to infection is <1)

Another Useful View of this Flow

- Recall: Total # of susceptibles infected per unit time = # of Susceptibles * "Likelihood" a given susceptible will be infected per unit time = S*(*"Force of Infection"*) = $S(c(I/N)\beta)$
- The above can also be phrased as the following: $S(c(I/N)\beta)=I(c(S/N)\beta)$=# of Infectives * Average # susceptibles infected per unit time by each infective
- This implies that as # of susceptibles falls=># of susceptibles surrounding each infective falls=>the rate of new infections falls ("Less fuel for the fire" leads to a smaller burning rate

Recall: The Importance of Susceptible Fraction

- Recall: Total # of susceptibles infected per unit time = # of Susceptibles * "Likelihood" a given susceptible will be infected per unit time = S*("*Force of Infection*") = $S(c(I/N)\beta)$
- The above can also be phrased as the following: $S(c(I/N)\beta)=I(c(S/N)\beta)$=# of Infectives * *Average # susceptibles infected per unit time by each infective*
- This implies that as Fraction of susceptibles falls=>Fraction of susceptibles surrounding each infective falls=>the rate of new infections falls ("Less fuel for the fire" leads to a smaller burning rate)