Infectious Disease Models 5 -- Vaccination

CMPT 858
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3-30-2010

Recall: Thresholds

- R*
  - Too low # susceptibles => R* < 1: # of infectives declining
  - Too high # susceptibles => R* > 1: # of infectives rising
- Outflow from susceptibles (infections) is determined by the # of Infectives
- Delays:
  - For a while after infectives start declining, they still deplete susceptibles sufficiently for susceptibles to decline
  - For a while after infectives start rising, the # of infections is insufficient for susceptibles to decline

Effective Reproductive Number: R_*

- Number of individuals infected by an ‘index’ infective in the current epidemiological context
- Depends on
  - Contact number
  - Transmission probability
  - Length of time infected
  - # (Fraction) of Susceptibles
- Affects
  - Whether infection spreads
    - If $R_* > 1$, # of cases will rise, If $R_* < 1$, # of cases will fall
      - Alternative formulation: Largest real eigenvalue <> 0
  - Endemic Rate

Basic Reproduction Number: R_0

- Number of individuals infected by an 'index' infective *in an otherwise disease-free equilibrium*
  - This is just R_* at disease-free equilibrium all (other) people in the population are susceptible other than the index infective
- Depends on
  - Contact number
  - Transmission probability
  - Length of time infected
- Affects
  - Whether infection spreads
    - If $R_0 > 1$, Epidemic Takes off, If $R_0 < 1$, Epidemic dies out
      - Alternative formulation: Largest real eigenvalue <> 0
    - Initial infection rise $\propto$ exp(t*(R0-1)/D)
  - Endemic Rate

Recall: A Critical Throttle on Infection Spread: Fraction Susceptible (*f*)

- The fraction susceptible (here, S/N) is a key quantity limiting the spread of infection in a population
  - Recognizing its importance, we give this name *f* to the fraction of the population that issusceptible
- If contact patterns & infection duration remain unchanged and, then mean # of individuals infected by an infective over the course of their infection is $f*R_0$

Recall: Endemic Equilibrium

- Inflow=Outflow $\Rightarrow$ $(S/N)\cdot R_0 = f\cdot R_0 = 1$
  - Every infective infects a "replacement" infective to keep equilibrium
  - Just enough of the population is susceptible to allow this replacement
- The higher the R_0, the lower the fraction of susceptibles in equilibrium!
  - *Generally some* susceptibles remain: At some point in epidemic, susceptibles will get so low that can't spread

Equilibrium Behaviour

- With Births & Deaths, the system can approach an "endemic equilibrium" where the infection stays circulating in the population – but in balance
- The balance is such that (simultaneously)
  - The rate of new infections = The rate of immigration
    - Otherwise # of susceptibles would be changing!
  - The rate of new infections = the rate of recovery
    - Otherwise # of infectives would be changing!

Equilibria

- Disease free
  - No infectives in population
  - Entire population is susceptible
- Endemic
  - Steady-state equilibrium produced by spread of illness
  - Assumption is often that children get exposed when young
- The stability of the these equilibria (whether the system departs from them when perturbed) depends on the parameter values
  - For the disease-free equilibrium on R_0

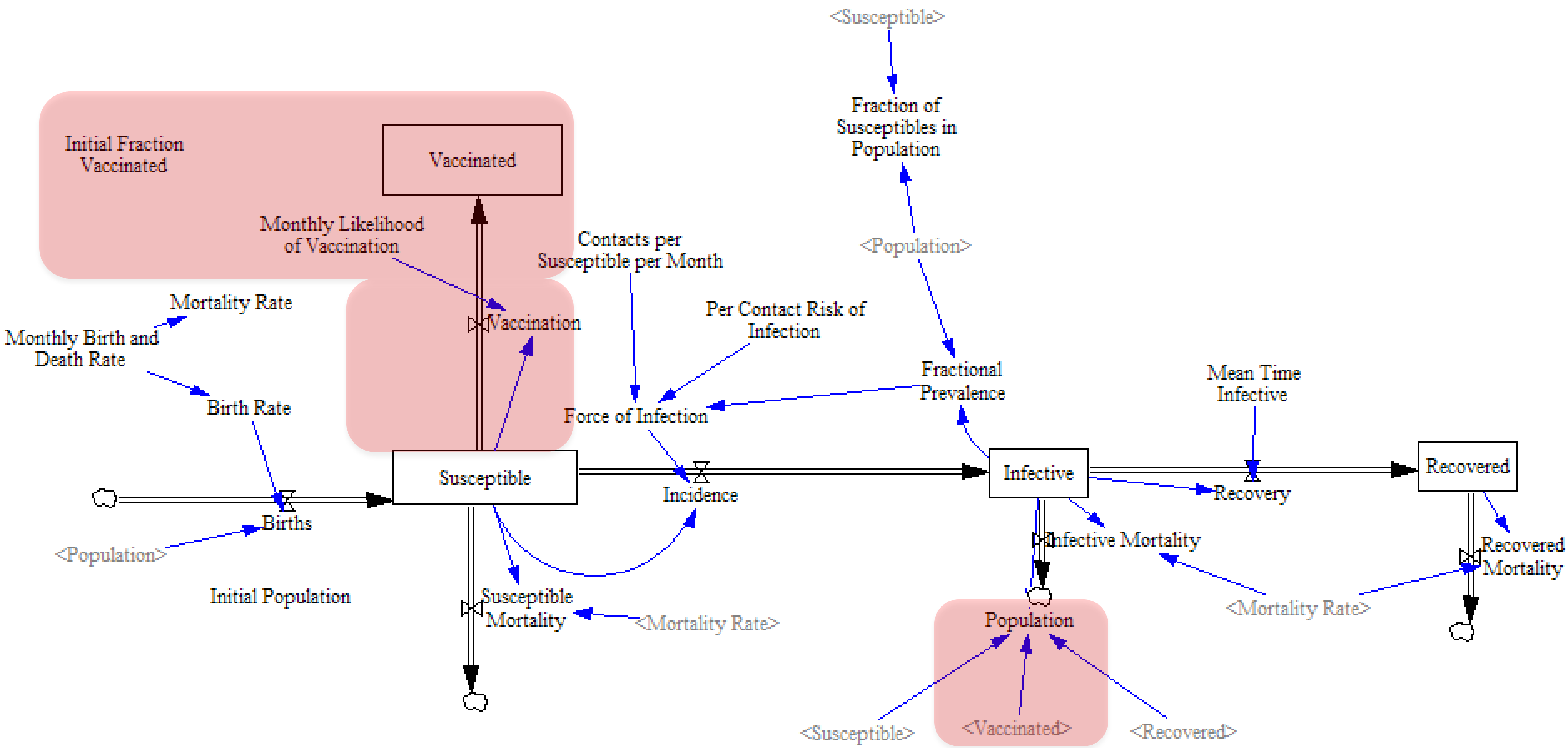
Adding Vaccination Stock

- Add a
  - "Vaccinated" stock
  - A constant called "Monthly Likelihood of Vaccination"
  - "Vaccination" flow between the "Susceptible" and "Vaccinated" stocks
    - The rate is the stock times the constant above
- Set initial population to be divided between 2 stocks
  - Susceptible
  - Vaccinated
- Incorporate "Vaccinated" in population calculation

Additional Settings

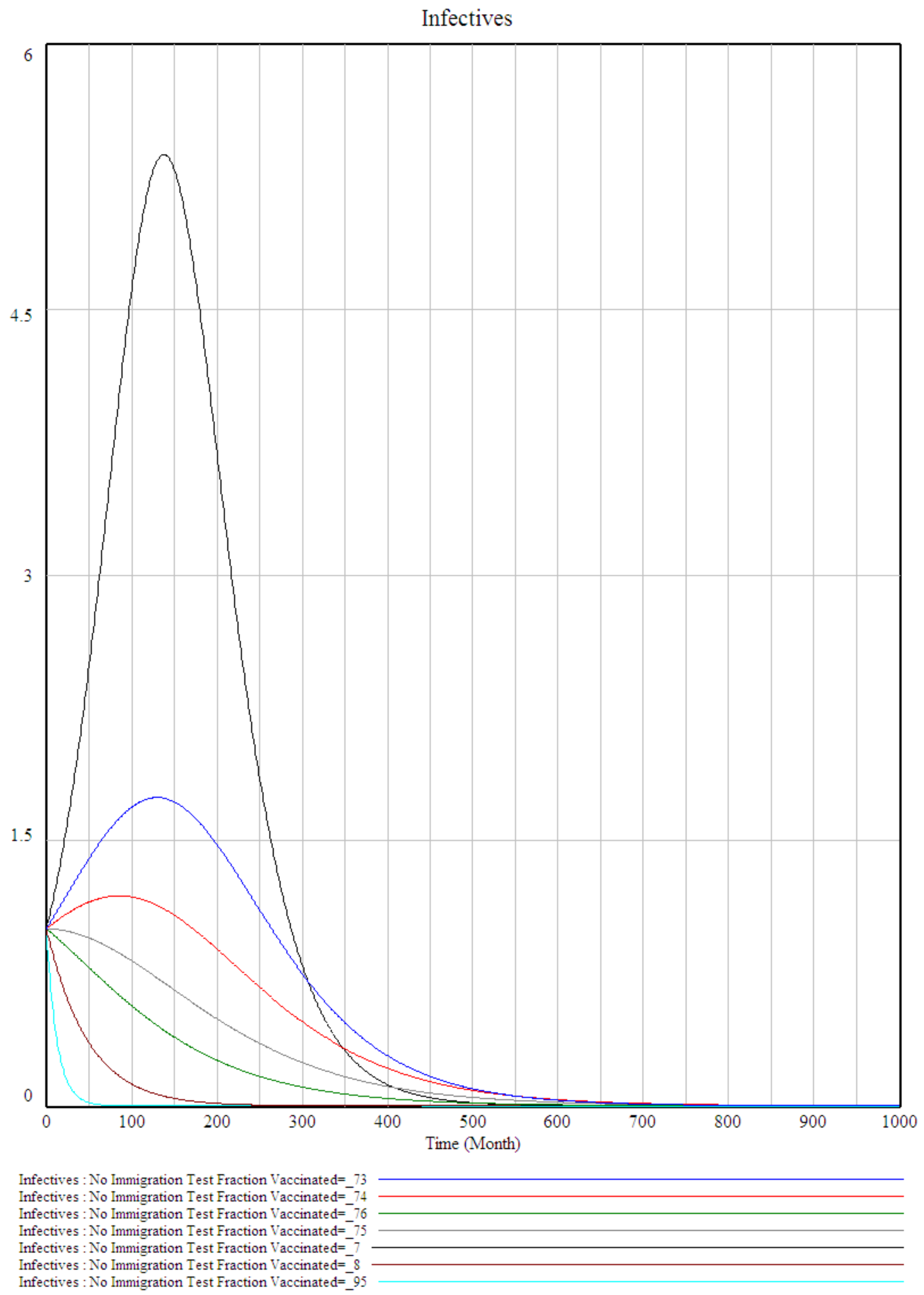
- c= 10
- Beta=.04
- Duration of infection = 10
- Birth & Death Rate=0

Adding Stock



Experiment with Different Initial Vaccinated Fractions

- Fractions = 0.25, 0.50, 0.6, 0.7, 0.8


Infectives
6
4.5
3
1.5
0
0
100
200
300
400
500
600
700
800
900
1000
Time (Month)
Infectives : No Immigration Test Fraction Vaccinated=_73
Infectives : No Immigration Test Fraction Vaccinated=_74
Infectives : No Immigration Test Fraction Vaccinated=_76
Infectives : No Immigration Test Fraction Vaccinated=_75
Infectives : No Immigration Test Fraction Vaccinated=_7
Infectives : No Immigration Test Fraction Vaccinated=_8
Infectives : No Immigration Test Fraction Vaccinated=_95

Critical Immunization Threshold

- Consider an index infective arriving in a “worst case” scenario when noone else in the population is infective or recovered from the illness
  - In this case, that infective is most “efficient” in spreading
- The goal of vaccination is keep the fraction susceptible low enough that infection cannot establish itself even in this worst case
  - We do this by administering vaccines that makes a person (often temporarily) immune to infection
- We say that a population whose f is low enough that it is resistant to establishment of infection exhibits “herd immunity”

Critical Immunization Threshold

- Vaccination seeks to lower *f* such that $f^{*}R_0<1$
- *Worst case: Suppose we have a population that is divided into immunized (vaccinated) and susceptible*
  - Let q_c be the critical fraction immunized to stop infection
  - *Then* $f=1-q_c$, $f^{*}R_0<1 \Rightarrow (1-q_c)^{*}R_0<1 \Rightarrow q_c>1-(1/R_0)$
- So if $R_0 = 4$ (as in our example), $q_c=0.75$ *(i.e. 75% of population must be immunized – just as we saw!)*