Infectious Disease Models 5: Intervention Impact on an Open Population

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Recall: The Importance of Susceptible Fraction

- Recall: Total # of susceptibles infected per unit time = # of Susceptibles * “Likelihood” a given susceptible will be infected per unit time = \( S \times ("Force of Infection") = S(c(I/N)\beta) \)

- The above can also be phrased as the following: \( S(c(I/N)\beta) = I(c(S/N)\beta) = \# \text{ of Infectives} \times \text{Average # susceptibles infected per unit time by each infective} \)

- This implies that as Fraction of susceptibles falls=>Fraction of susceptibles surrounding each infective falls=>the rate of new infections falls (“Less fuel for the fire” leads to a smaller burning rate)
A Critical Throttle on Infection Spread: Fraction Susceptible \((f)\)

- The fraction susceptible (here, \(S/N\)) is a key quantity limiting the spread of infection in a population.
  - Recognizing its importance, we give this name \(f\) to the fraction of the population that is susceptible.
- If contact patterns & infection duration remain unchanged and, then mean # of individuals infected by an infective over the course of their infection is \(f*R_0\).
Critical Immunization Threshold

• Consider an index infective arriving in a “worst case” scenario when no one else in the population is infective or recovered from the illness
  – In this case, that infective is most “efficient” in spreading

• The goal of vaccination is keep the fraction susceptible low enough that infection cannot establish itself even in this worst case
  – We do this by administering vaccines that makes a person (often temporarily) immune to infection

• We say that a population whose $f$ is low enough that it is resistant to establishment of infection exhibits “herd immunity”
Critical Immunization Threshold

• Vaccination seeks to lower \( f \) such that \( f*R_0 < 1 \)
• **Worst case:** Suppose we have a population that is divided into immunized (vaccinated) and susceptible
  
  – Let \( q_c \) be the critical fraction immunized to stop infection
  
  – *Then* \( f = 1 - q_c, f*R_0 < 1 \) \( \Rightarrow (1-q_c)*R_0 < 1 \) \( \Rightarrow q_c > 1 - (1/R_0) \)

• So if \( R_0 = 4 \) (as in our example), \( q_c = 0.75 \) (i.e. 75% of population must be immunized – just as we saw!)
## Open/Closed Population

<table>
<thead>
<tr>
<th>Case</th>
<th>Epidemic Occurs?</th>
<th>Steady-state Fraction infective</th>
<th>Steady-state Fraction susceptible</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Open Population</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_0 &gt; 1$</td>
<td>Yes</td>
<td>Such that Infection rate=Recovery rate</td>
<td>$1/R_0$</td>
</tr>
<tr>
<td>$R_0 &lt; 1$</td>
<td>No</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Closed Population</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_0 &gt; 1$</td>
<td>Yes</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$R_0 &lt; 1$</td>
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</tbody>
</table>
Effects of An Open Population (different Parameters)

Approaches endemic level where $R_*=1$ & rate of new infections = rate of recoveries

Because no new influx of susceptibles ("fuel"), infectives in constant Decline. Approaches 0 (disease-free equilibrium)
Effects of An Open Population

Susceptible

Approaches endemic level where $R_*=1$
& rate of arrivals (via birth&migration) = rate of new infections+deaths

Approaches disease-free level where no infection is occurring
Recovered : Baseline 2% Annual Turnover
Recovered : Baseline Closed Population
Impact of Turnover

• The greater the turnover rate, the greater the fraction of susceptibles in the population => the greater the endemic rate of infection
Effective Reproductive Number
Fraction Recovered

Fraction of Recovereds in the Population

Time (Year)

Fraction of Recovereds in the Population: Baseline 20% Population Turnover
Fraction of Recovereds in the Population: Baseline 10% Population Turnover
Fraction of Recovereds in the Population: Baseline 5% Population Turnover
Fraction of Recovereds in the Population: Baseline 2% Population Turnover
Fraction of Recovereds in the Population: Baseline 1% Population Turnover
Fraction of Recovereds in the Population: Baseline No Population Turnover
Adding Ongoing Vaccination Process
Simulating Introduction of Vaccination for a Childhood Infection in an Open Population

- $c = 500$
- Beta = 0.05
- Duration of infection = 0.25
- Initial Fraction Vaccinated = 0
- Monthly birth & death rate = 10% per year
  (focusing on children 0-10 years of age)

Questions
- What is $R_0$?
- What level of susceptibles is required to sustain the infection
- What is the critical vaccination fraction?
Fraction of Population Vaccinated

Time (Year)

Fraction of Population Vaccinated: Vaccination 50% per year 10% Population Turnover
Fraction of Population Vaccinated: Vaccination 40% per year 10% Population Turnover
Fraction of Population Vaccinated: Vaccination 20% per year 10% Population Turnover
Fraction of Population Vaccinated: Vaccination 10% per year 10% Population Turnover
What Rate of Vaccination Eliminates?
Representing Quarantine
Endemic Situations

• In an endemic context, infection remains circulating in the population

• The common assumption here is that
  • The susceptible portion of the population will be children
  • At some point in their life trajectory (at an average age of acquiring infection A), individuals will be exposed to the infection & develop immunity
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Age of Exposure & Reproductive Constant

• Cf a “natural” (non-immunized) constant size population where all die at same age and where
  – Mean Age at death L
  – Mean Age of exposure A (i.e. we assume those above A are exposed)

• Fraction susceptible is \( \frac{S}{N} = \frac{A}{L} \) (i.e. proportion of population below age A)

• Recall for our (and many but not all other) models:
  \( R^* = (S/N)R_0 = 1 \implies \frac{S}{N} = \frac{1}{R_0} \)

• Thus
  \[
  \frac{A}{L} = \frac{1}{R_0} \implies \frac{L}{A} = R_0
  \]

  • This tells us that the larger the \( R_0 \), the earlier in life individuals become infected
Incompletely Immunized Population

- Suppose we have $q$ fraction of population immunized ($q < q_c$)
- Suppose we have fraction $f$ susceptible
- Fraction of the population currently or previously infected is $1 - q - f$
  - If we assume (as previously) that everyone lives until $L$ and is infected at age $A$, then fraction $1 - A/L$ has been infected
  - So $1 - A/L = 1 - q - f \implies A = L(q + f)$
    - This can be much higher than for the natural population
      - This higher age of infection can cause major problems, due to waning of childhood defenses
    - i.e. incomplete immunization leads to older mean age of exposure
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