Uncertainty, Stochastics & Sensitivity Analysis

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CMPT 858

March 17, 2011
Types of Sensitivity Analyses

• Variables involved
  – One-way
  – Multi-way

• Type of component being varied
  – Parameter sensitivity analysis: Parameter values
  – Structural sensitivity analysis: Examine effects of model structure on results

• Type of variation
  – Single alternative values
  – Monte Carlo analyses: Draws from probability distributions (many types of variations)

• Frequency of variation
  – Static (parameter retains value all through simulation)
  – Ongoing change: Stochastic process
    • Accomplished via Monte-Carlo analyses
    • Key for DES & ABM
Model Uncertainty

• Here, we are frequently examining the impact of changing
  – Our assumptions about “how the system works”
  – Our decision of how to abstract the system behaviour

• Structural sensitivity analyses
  – Vary structure of model & see impact on
    • Results
    • Tradeoffs between choices
  – Frequently recalibrate the model in this process

• Here, we are considering uncertainty about how the current state is mapped to the next state
Predictor-Corrector Methods: Dealing with an Incomplete Model

• Some approaches (e.g. Kalman filter, Particle Filter) are motivated by awareness that models are incomplete

• Such approaches try to adjust model state estimates on an ongoing basis,
  – Given uncertainty about model predictions
  – New observations

• Assumption here is that the error in the model is defined by some probability distribution
Static Uncertainty
Sensitivity Analyses

• In variation, one can seek to investigate different
  – Assumptions
  – Policies
• Same relative or absolute uncertainty in different parameters may have hugely different effect on outcomes or decisions
• Help identify parameters/initial states that strongly affect
  – Key model results
  – Choice between policies
• We place more emphasis in parameter estimation & interventions into parameters exhibiting high sensitivity
Spider Diagram

• Each axis represents a % change in a particular parameter
  – This proportional change is identical for the different parameters

• The distance assumed by the curve along that axis represents the magnitude of response to that change
  – Note that these sensitivities will depend on the state of system!

http://www.niwotridge.com/images/BLOGImages/SpiderDiagram.jpg
Systematic Examination of Policies

Tengs, Osgood, Lin
Sensitivity Analyses in Vensim

Sensitivity Control. Edit the filename to save changes to a different control file

Filename: Monte Carlo SIR.vsc

Number of simulations: 20000
Seed: 1234

- Display warning messages
- Currently active parameters (drag to reorder)
- Noise Seed = RANDOM_UNIFORM(0, 10000)

Parameter: Mean Time to Recover
Value: VECTOR

Model Value | Minimum Value | Maximum Value | Increment
---|---|---|---
1 | 0 | 10 | 1

Options:
- Multivariate
- Univariate
- Latin Hypercube
- Latin Grid
- File

Buttons:
- Delete Selected
- Modify Selected
- Add Editing
- OK
- Cancel
Sensitivity in Initial States

• Frequently we don’t know the exact state of the system at a certain point in time
• A very useful type of sensitivity analysis is to vary the initial model state
• In Vensim, this can be accomplished by
  – Indicating a parameter name within the “initial value” area for a stock
  – Varying the parameter value
• In an agent-based model, state has far larger dimensionality
  – Can modify different numbers of people with characteristic, location of people with characteristic, etc.
Imposing a Probability Distribution
Monte Carlo Analysis

• We feed in probability distributions to reflect our uncertainty about one or more parameters
• The model is run many, many times (realizations)
  – For each realization, the model uses a different draw from those probability distribution
• What emerges is resulting probability distribution for model outputs
Example Resulting Distribution

Empirical Fractiles
Multi-Way Sensitivity Analyses

• When examining the results of changing multiple variables, need to consider how multiple variables vary together

• If this covariation reflects dependence on some underlying factor, may be able to simulate uncertainty in underlying factor
Performing Monte Carlo Sensitivity Analyses in Vensim

• Need to specify three things
  – The parameters to vary
  – How to vary those parameters
  – Which model variables to save away
How & What Parameters to Vary

Sensitivity Control. Edit the filename to save changes to a different control file.

Filename: Simple SIR.vsc

- Number of simulations: 1000
- Noise Seed: 1234

- Display warning messages

Currently active parameters (drag to reorder):

Annual Birth and Death Rate = RANDOM_NORMAL(0.05, 0.02, 0.01)

Parameter: Per Contact Risk of Infection
Value: 0.05
Minimum Value: 0.02
Maximum Value: 0.1

Distribution: RANDOM_UNIFORM

Buttons:
- Clear Settings
- Choose New File...
- Add Editing
- Modify Selected
- Delete Selected
- OK
- Cancel
Monte Carlo Analyses

Sensitivity Simulation 483/1000

Fraction of Population Vaccinated

Annual Likelihood of Vaccination

Rate

Births

Population

Susceptible

Force of Infection

Incidence

Infective

Contact Rate

Quarantine

Death

Population

Infective Mortality

Mortality Rate

Contacts per Susceptible per Year

Fraction of Susceptibles in Population

Time to Acquire Infection

Incidence
Sensitivity Results (Prevalence)
An observation at this point in time would produce a histogram (approximating a distribution) for fraction of susceptibles.
Stochastic Processes

• System Dynamics models are traditionally deterministic

• As will be discussed in tutorials, models can be made stochastic
  – Transitions between states (e.g. duration of infection or immunity)
  – Transmission of infection

• As a result, there will be variation in the results from simulation to simulation
Summarizing Variability

• To gain confidence in model results, typically need to run an ensemble of realizations
  – Deal with means, standard deviations, and empirical fractiles
  – As is seen here, there are typically still broad regularities between most runs (e.g. rise & fall)

• Need to reason over a population of realizations
  ⇒ statistics are very valuable
  – Fractile within which historic value falls
  – Mean difference in results between interventions
Closing Question: How can we best adapt our policies to deal with ongoing uncertainty?

- We are dealing here with making decisions in an environment that changes over time.
- This uncertainty could come from:
  - Stochastic variability
  - Uncertainty regarding parameter values
- There is an incredibly vast # of possible policies.
Stochastic Processes in Vensim