Causal Loop Wrap-Up & Stocks-Flow

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Feedbacks Driving Infectious Disease Dynamics

- Susceptibles
- Contacts between Susceptibles and Infectives
- Infectives
- New Infections
- New Recoveries

+ + +
- - -

+ + +
- - -

+ + +
- - -

+ + +
- - -
Example Dynamics of SIR Model (No Births or Deaths)

Susceptible Population S : SIR example people
Infectious Population I : SIR example people
Recovered Population R : SIR example people
Shifting Feedback Dominance

SIR Example

Susceptible Population S : SIR example
Infectious Population I : SIR example
Recovered Population R : SIR example
Issues with Causal Loop Diagrams

- Unclear variables
- Diagrams can become very large
- Confusion regarding polarity
- Non-causal relationship
- Conservation not captured
- Behavior not always same as archetype
- Missing causal factors
- Missing links
- Asymmetry in direction of change
Unclear Variables

Variables Lacking Clear Polarity

• Gender
• Ethnicity
• Shape

*Often categorical & non-ordinal*

• Ask whether “more X” is
  – Meaningful
  – Unambiguous

Implicit Polarity

• Population (size)
• Revenue (amount of)
• Sound, Color (more of)
• Socioeconomic status (greater, lesser)
Very Large Diagrams

Still useful for getting “big picture” identifying where research “fits in”, research gaps

http://kim.foresight.gov.uk/Obesity/Obesity.html
Artifactual Loop 1
Artifactual Loop

Fraction of the Population that is Diabetic

Fraction of the Population that is Non Diabetic
Artifactual Loop 2
Introduction to Stocks & Flows

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State of the System: Stocks (“Levels”, “State Variables”, “Compartments”)

- Stocks (Levels) represent accumulations
  - These capture the “state of the system”
  - Mathematically, we will call these “state variables”
- These can be measured at one instant in time
- Stocks start with some initial value & are thereafter changed only by flows into & out of them
  - There are no inputs that immediately change stocks
- Stocks are the source of delay in a system
- In a stock & flow diagram, shown as rectangles
Examples of Stocks

- Water in a tub or reservoir
- People of different types
  - {Susceptible, infective, immune} people
  - Pregnant women
  - Women between the age of $x$ and $y$
  - High-risk individuals
- Healthcare workers
- Medicine in stocks
- Money in bank account
- CO$_2$ in atmosphere
- Blood sugar
- Stored Energy
- Degree of belief in X
- Stockpiled vaccines
- Goods in a warehouse
- Beds in an emergency room
- Owned vehicles
Example Model: Stocks
The Critical Role of Stocks in Dynamics

- Stocks determine current state of system
  - Stocks often provide the basis for making choices
- Stocks central to most disequilibria phenomena (buildup, decay)
- Lead to inertia
- Give rise to delays
State Changes: Flows ("Fluxes", "Rates", "Derivatives")

- All changes to stocks occur via flows

- Always expressed per some unit time: If these flow into/out of a stock that keeps track of things of type $X$ (e.g. persons), the rates are measured in $X/(\text{Time Unit})$ (e.g. persons/year, $/\text{month}$, gallons/second)

- Typically measure over certain period of time (by considering accumulated quantity over a period of time)
  - e.g. Incidence Rates is calculated by accumulating people over a year, revenue is $/\text{Time}$, water flow is litres/minute
  - Can be estimated for any point in time
Examples of Flows

- Inflow or outflow of a bathtub (litres/minute)
- Rate of incident cases (e.g. people/month)
- Rate of recovery
- Rate of mortality (e.g. people/year)
- Rate of births (e.g. babies/year)
- Rate of treatment (people/day)
- Rate of caloric consumption (kcal/day)
- Rate of pregnancies (pregnancies/month)
- Reactivation Rate (# of TB cases reactivating per unit time)
- Revenue ($/month)
- Spending rate ($/month)
- Power (Watts)
- Rate of energy expenditure
- Vehicle sales
- Vaccine sales
- Shipping rate of goods
Example Model: Flows

Mean Contacts Per Susceptible per Year

Mean Infectious Contacts Per Susceptible per Year

Total Population

Force of Infection (Likelihood Density of Infection per Susceptible)

Prevalence of Infection

Recovery Delay

Loss of Immunity Delay

Initial Population

Cumulative Illnesses

New Infections

New Recovery

Temporarily Immune

Newly Susceptible
Flows 2

• We can ask conceptually *about* the rate *at* any given point in time – and may change over time
  – Measuring it would have to be over some period

• When speaking about “rates” for flows, we always mean a *rate of change over time* (something measured as *X/Unit Time*)
  – Not all things called “rates” are flows
    • Exchange rate
    • Prevalence rate
    • Rate of return
Distinguishing Stocks & Flows: Heuristics

• To determine if a quantity is a stock or flow:
  – “Snapshot” test: If you were only to consider a moment in time (a “snapshot” of the system), could the quantity be clearly quantified by the information available at that moment?
    • If yes, stock (cannot quantify a value of a flow using only the information for an instant – must measure over time)
  – “Time unit change” test: If we were to change the unit by which we measure time, would the numeric value of the quantity change?
    • If yes, quite likely to be a flow (exception: beliefs about flows)
  – “Accumulation” test: Is this quantity an accumulation of the time-varying values of other quantities?
    • If yes, stock
Exercise: Stocks or Flows?

- Account balance
- Income
- Incidence
- Prevalence
- Temperature
- Births
- Profits
- Interest
- Principal
- Shipments
- Car accidents
- Patients on dialysis
- Deaths
- Heart attacks
- Arrests
- Police
- Patients in hospital
- Hospital admissions
- Position
- Speed
Key Component: Stock & Flow

Flow -> Stock

Flow

Stock
Net Flow Impact on Stock

Impact of Lowering Flow (Rate) to 5/Month?
Loops & Stocks

• Causation does not effect big change instantaneously
  – Loops are not instantaneous

• Stocks only change by changes to the flows into & out of them
  – There are no inputs that immediately change stocks

• All causal loops must involve at least one stock
  – The state of the world must change as part of the process
  – Absent a stock, loop would be instantaneous
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Auxiliary Variables

• Auxiliary variables are convenience names we give to concepts that can be defined in terms of expressions involving stocks/flows at current time
  – Adding or eliminating an auxiliary variable does not change the mathematical structure of the system

• Critical for model transparency
  – Can be reused at many places
  – References to auxiliary variables prevents need for modeler to think about all of details of definition

• Enhanced modifiability: Single place to define

• Convenient for reporting (graphing, tables) & analyzing model dynamics
Example Model: Auxiliary Variables

- Mean Contacts Per Susceptible per Year
- Force of Infection (Likelihood Density of Infection per Susceptible)
- Probability of Transmission between Infective and Susceptible
- Initial Population
- New Infections
- New Recovery
- Loss of Immunity Delay
- Newly Susceptible
- Cumulative Illnesses
- New Illnesses
- Total Population
- Prevalence of Infection
- Recovery Delay
- Temporarily Immune
For similar reasons to auxiliary variables, we give names to:

- Model constants
- Time series
Example Model: Parameters
Stocks & Flows Compared with Markov Models

• Open population
  – Births
  – Deaths

• Non-constant likelihood (density) of transitions
  – Likelihood of leaving a stock per unit time can depend on other stocks
    • Force of Infection (likelihood of susceptible becoming infected) can depend on prevalence of illness
    • Likelihood of initiating smoking could depend on accumulated current or former smokers

• Multiple types of stocks
  – e.g. costs, QALYs, hosts & reservoir species, etc.

• Continuous time
Distinctive Stock & Flow Features

- Mean Contacts Per Susceptible per Year
- Mean Infectious Contacts Per Susceptible per Year
- Total Population
- Prevalence of Infection
- Recovery Delay
- Loss of Immunity Delay
- Newly Susceptible
- New Infections
- New Recovery
Multi-Species Model (West Nile Virus)
Refinement of Causal Loop Diagrams: System Structure Diagrams

• Still essentially a qualitative model, but less ambiguous
  – By clearly distinguish stocks & flows, this helps reduce the artifactual loops discussed with CLDs

• Combine causal loops diagram elements with stock & flow structure

• If complete, all loops will go “through a stock”
  – Loop goes into the flow of a stock (as one variable in the diagram)
  – Loop comes comes out of stock (as next variable in diagram)
Example System Structure Diagram

Note treatment of flows as links from flow to stock
• Inflows as positive links
• Outflows as negative links
Stocks & Flows: Diabetes

• Assume diabetes is not curable

• Stocks:
  – People without diabetes (at different stages of risk?)
  – People with diabetes

• Flows
  – Incident cases (both diagnosed & undiagnosed!)
  – Deaths from both stocks
Stocks & Flows: Tuberculosis

- Assume that TB infection cannot be totally eliminated
- Stocks
  - Susceptible people
  - Immunized people
  - People with latent TB infection
  - People with active TB infection
- Flows
  - People becoming latently infected
  - People being vaccinated
  - People with infection going to Active TB ("primary progression")
  - People with infection going on to latent TB
  - People with secondary infection going on to active TB
  - Deaths from each stock
Diabetes Model Stocks & Flows
(For a Challenge, Try Creating this in Vensim!)

People without Diabetes

People with Diabetes

Incident cases of Diabetes

Deaths of People with Diabetes

Deaths of People without Diabetes

Use Initial Value: 1000

Use Value: 0

Use Value: 10

Use Value: 15
Interactive Steps

• View flows and stocks – does this make sense?
• Hitch up constant “auxiliary” variables to flows
• How does changing constant variables change the stock?
What happens to the stock?
Resulting Stock (Green)

Diabetes Stock & Flows

Deaths of People with Diabetes: Current
Incident cases of Diabetes: Current
People with Diabetes: Current
Suppose we have these Flows (Rates)

Deaths of People with Diabetes: Stock and Flow Demonstration Test

Incident cases of Diabetes: Stock and Flow Demonstration Test

What happens to the stock?
Some Questions

• When is the stock of people with diabetes at its lowest value?
• When is the stock of people with diabetes at its greatest value?
• Is the value of the stock of people with diabetes larger at the beginning or end?
• When is the stock of people with diabetes not changing?
Flows and Feedbacks

- Stocks are always changed by flows
- In your experiments, we’ve used constant values for flows
- In general, the formulas for the flows will depend on things that are changing (state)
  - Ultimately, these things must depend on the things that collectively specify the state – the stocks!
Stocks As Accumulations

- We often use stocks to accumulate (integrate) other (evolving) quantities over time.

- Example (assume time measured in years):

  A Key Reflection: If we have population of size $P$, after 1 year, the stock holds $1*P$. After 2 years, $2*P$, after $n$ years, $n*P$. With a changing $P$, this integrates $P$ over time.
Example 2

Patients in Sanatoria

New Sanatoria Patient Years

Cumulative Sanatoria Patient Years
Slightly more Sophisticated

Quality of Life for subpopulation 2

Quality Weighted subpopulation 2

[Formula: Quality of Life for subpopulation 2 * Size of subpopulation 2]

Size of subpopulation 2

Quality Weighted Entire Population

[Formula: Quality Weighted subpopulation 1 + Quality Weighted subpopulation 2]

New Life Years Lived

Quality Weighted subpopulation 1

Quality of Life for subpopulation 1

[Formula: Quality of Life for subpopulation 1 * Size of subpopulation 1]

Size of subpopulation 1

Cumulative Quality Adjusted Life Years Lived
Principle: Structure Determines Behaviour

• Feedback & stock-and-flow structure of a system determines the possible patterns of behaviour
• Different sets of parameters (e.g. values for constants) will select particular behaviour within these behaviour patterns
• Changes to the feedback structure can change behaviour in fundamental ways
Simple SIT Model

Mean Contacts Per Capita

Per infected contact infection rate

Initial Population

Mean Infectious Contacts Per Susceptible

Per Susceptible Incidence Rate

Prevalence

Recovery Delay

Total Population

Mean Contacts

New infections

Newly Susceptible

Immunity loss

Delay

Incidence Rate

Cumulative Illnesses

New Illness

Prevalence

Recovery Delay

Initial Population

S

I

R

New infections

New Recovery

Newly Susceptible

Immunity loss Delay
Classic Feedbacks

Susceptibles

Contacts of Susceptibles with Infectives

Infectives

New Infections
Dynamics

State variables over time

S : Alternative 30 HC Workers Exogenous Recovery Delay
I : Alternative 30 HC Workers Exogenous Recovery Delay
R : Alternative 30 HC Workers Exogenous Recovery Delay
Broadening the Model Boundaries: Endogenous Recovery Delay

- Mean Contacts Per Capita
- Mean Infectious Contacts Per Susceptible
- Per Susceptible Incidence Rate
- Fractional Prevalence
- Recovery Delay
- Staff Time per Patient
- Total Population
- Healthcare Workers
- Time Until Seek Treatment
- Initial Population
- New infections
- New Susceptible
- New Recovery
- Newly Susceptible
- Immunity loss Delay

- Per infected contact infection rate
Broadening the Model Boundaries: Endogenous Recovery Delay

Susceptibles

Contacts of Susceptibles with Infectives

Infectives

New Infections

People Presenting for Treatment

Waiting Times

Health Care Staff

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A Different Behaviour Mode

Prevalence, Infectious

Time (Day)

Prevalence : Baseline 30 HC Workers
I : Baseline 30 HC Workers

0 1 Person
0 100,000 Person
0 200,000 Person
Structure as Shaping Behaviour

• System structure is defined by
  – Stocks
  – Flows
  – Connections between them

• Nonlinearity: The behaviour of the whole is more than the sum of the behaviour of the parts
  – “Emergent” behaviour would not be anticipated from simple behaviour of each piece in turn

• Stock and flow structure (including feedbacks) of a system determines the qualitative behaviour modes that the system can take on