Introduction to Stocks & Flows

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CMPT 858
State of the System: Stocks ("Levels", "State Variables", "Compartments")

- Stocks (Levels) represent accumulations
  - These capture the "state of the system"
  - Mathematically, we will call these "state variables"
- These can be measured at *one instant in time*
- Stocks start with some initial value & are thereafter changed only by *flows* into & out of them
  - There are no inputs that immediately change stocks
- Stocks are the source of delay in a system
- In a stock & flow diagram, shown as *rectangles*
Examples of Stocks

• Water in a tub or reservoir
• People of different types
  – {Susceptible, infective, immune} people
  – Pregnant women
  – Women between the age of x and y
  – High-risk individuals
• Healthcare workers
• Medicine in stocks

• Money in bank account
• CO₂ in atmosphere
• Blood sugar
• Stored Energy
• Degree of belief in X
• Stockpiled vaccines
• Goods in a warehouse
• Beds in an emergency room
• Owned vehicles
Example Model: Stocks
The Critical Role of Stocks in Dynamics

• Stocks determine current state of system
  – Stocks often provide the basis for making choices

• Stocks central to most disequilibria phenomena (buildup, decay)

• Lead to inertia

• Give rise to delays
State Changes: Flows ("Fluxes", "Rates", "Derivatives")

• All changes to stocks occur via flows

• Always expressed per some unit time: If these flow into/out of a stock that keeps track of things of type $X$ (e.g. persons), the rates are measured in $X/(\text{Time Unit})$ (e.g. persons/year, $/\text{month}$, gallons/second)

• Typically measure over certain period of time (by considering accumulated quantity over a period of time)
  – e.g. Incidence Rates is calculated by accumulating people over a year, revenue is $/\text{Time}$, water flow is litres/minute
  – Can be estimated for any point in time
Examples of Flows

- Inflow or outflow of a bathtub (litres/minute)
- Rate of incident cases (e.g. people/month)
- Rate of recovery
- Rate of mortality (e.g. people/year)
- Rate of births (e.g. babies/year)
- Rate of treatment (people/day)
- Rate of caloric consumption (kcal/day)

- Rate of pregnancies (pregnancies/month)
- Reactivation Rate (# of TB cases reactivating per unit time)
- Revenue ($/month)
- Spending rate ($/month)
- Power (Watts)
- Rate of energy expenditure
- Vehicle sales
- Vaccine sales
- Shipping rate of goods
Example Model: Flows

- Mean Contacts Per Susceptible per Year
- Mean Infectious Contacts Per Susceptible per Year
- Total Population
- Force of Infection (Likelihood Density of Infection per Susceptible)
- Prevalence of Infection
- Recovery Delay
- New Infections
- New Recovery
- Loss of Immunity Delay
- Newly Susceptible
- Initial Population
- New Illnesses
- Cumulative Illnesses
- Temporarily Immune
Key Component: Stock & Flow

Flow → Stock

Stock

Stock

Flow

+
Net Flow Impact on Stock

Impact of Lowering Flow (Rate) to 5/Month?
Loops & Stocks

• Causation does not effect big change instantaneously
  – Loops are not instantaneous

• Stocks only change by changes to the flows into & out of them
  – There are no inputs that immediately change stocks

• All causal loops must involve at least one stock
  – The state of the world must change as part of the process
  – Absent a stock, loop would be instantaneous
Auxiliary Variables

• Auxiliary variables are convenience names we give to concepts that can be defined in terms of expressions involving stocks/flows at current time
  – Adding or eliminating an auxiliary variable does not change the mathematical structure of the system
• Critical for model transparency
  – Can be reused at many places
  – References to auxiliary variables prevents need for modeler to think about all of details of definition
• Enhanced modifiability: Single place to define
• Convenient for reporting (graphing, tables) & analyzing model dynamics
Example Model: Auxiliary Variables
Constants & Time Series Parameters

• For similar reasons to auxiliary variables, we give names to
  – Model constants
  – Time series
Example Model: Parameters

- Mean Contacts Per Susceptible per Year
- Force of Infection (Likelihood Density of Infection per Susceptible)
- Probability of Transmission between Infective and Susceptible
- Initial Population
- Mean Infectious Contacts Per Susceptible per Year
- Total Population
- Prevalence of Infection
- Recovery Delay
- Loss of Immunity Delay
- New Illnesses
- Newly Susceptible
- Temporarily Immune
- New Infections
- New Recovery
Stocks & Flows Compared with Markov Models

• Open population
  – Births
  – Deaths

• Non-constant likelihood (density) of transitions
  – Likelihood of leaving a stock per unit time can depend on other stocks
    • Force of Infection (likelihood of susceptible becoming infected) can depend on prevalence of illness
    • Likelihood of initiating smoking could depend on accumulated current or former smokers

• Multiple types of stocks
  – e.g. costs, QALYs, hosts & reservoir species, etc.

• Continuous time
Distinctive Stock & Flow Features

- Mean Contacts Per Susceptible per Year
- Mean Infectious Contacts Per Susceptible per Year
- Total Population
- Force of Infection (Likelihood Density of Infection per Susceptible)
- Probability of Transmission between Infective and Susceptible
- Initial Population
- New Infections
- New Recovery
- Loss of Immunity Delay
- Newly Susceptible
- Temporarily Immune

New Illnesses

Cumulative Illnesses
Multi-Species Model (West Nile Virus)
Refinement of Causal Loop Diagrams: System Structure Diagrams

• Still essentially a qualitative model, but less ambiguous
  – By clearly distinguish stocks & flows, this helps reduce the artifactual loops discussed with CLDs

• Combine causal loops diagram elements with stock & flow structure

• If complete, all loops will go “through a stock”
  – Loop goes into the flow of a stock (as one variable in the diagram)
  – Loop comes out of stock (as next variable in diagram)
Example System Structure Diagram

Note treatment of flows as links from flow to stock
• Inflows as positive links
• Outflows as negative links
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Stocks & Flows: Diabetes

• Assume diabetes is not curable
• Stocks:
  – People without diabetes (at different stages of risk?)
  – People with diabetes
• Flows
  – Incident cases (both diagnosed & undiagnosed!)
  – Deaths from both stocks
Stocks & Flows: Tuberculosis

• Assume that TB infection cannot be totally eliminated
• Stocks
  – Susceptible people
  – Immunized people
  – People with latent TB infection
  – People with active TB infection
• Flows
  – People becoming latently infected
  – People being vaccinated
  – People with infection going to Active TB (“primary progression”)
  – People with infection going on to latent TB
  – People with secondary infection going on to active TB
  – Deaths from each stock
Diabetes Model Stocks & Flows
(For a Challenge, Try Creating this in Vensim!)

- People without Diabetes
- People with Diabetes
- Incident cases of Diabetes
- Deaths of People with Diabetes
- Deaths of People without Diabetes

Use Initial Value: 1000
Use Value: 0
Use Value: 15
Use Value: 10
Interactive Steps

• View flows and stocks – does this make sense?
• Hitch up constant “auxiliary” variables to flows
• How does changing constant variables change the stock?
What happens to the stock?
Suppose we have these Flows (Rates)

Deaths of People with Diabetes: Stock and Flow Demonstration Test
Incident cases of Diabetes: Stock and Flow Demonstration Test

What happens to the stock?
Some Questions

• When is the stock of people with diabetes at its lowest value?
• When is the stock of people with diabetes at its greatest value?
• Is the value of the stock of people with diabetes larger at the beginning or end?
• When is the stock of people with diabetes not changing?
Flows and Feedbacks

• Stocks are always changed by flows
• In your experiments, we’ve used constant values for flows
• In general, the formulas for the flows will depend on things that are changing (state)
  – Ultimately, these things must depend on the things that collectively specify the state – the stocks!
Example 2

Patients in Sanatoria

New Sanatoria Patient Years

Cumulative Sanatoria Patient Years
Simple First-Order Decay
(Create this in Vensim!)

Use Initial Value: 1000

Use Formula: Deaths from Infection / Mean time until Death
Set Model Settings (Model Menu/Settings Item)

NOTE: To change later use Model>Settings or edit the equations for the above parameters.
Dynamics of Stock?
Dynamics of (Rate of) Death Flow?
Stocks As Accumulations

- We often use stocks to accumulate (integrate) other (evolving) quantities over time.

- Example (assume time measured in years):
  
  A Key Reflection: If we have population of size \( P \), after 1 year, the stock holds \( 1 \times P \). After 2 years, \( 2 \times P \), after \( n \) years, \( n \times P \).

  With a changing \( P \), this integrates \( P \) over time.
Example 2

Patients in Sanatoria

New Sanatoria Patient Years

Cumulative Sanatoria Patient Years
Principle: Structure Determines Behaviour

• Feedback & stock-and-flow structure of a system determines the possible patterns of behaviour
• Different sets of parameters (e.g. values for constants) will select particular behaviour within these behaviour patterns
• Changes to the feedback structure can change behaviour in fundamental ways
Simple SIT Model

- Mean Contacts Per Capita
- Per infected contact infection rate
- Initial Population
- Mean Infectious Contacts Per Susceptible
- Per Susceptible Incidence Rate
- Prevalence
- Recovery Delay
- Total Population
- Mean Contacts Per Susceptible
- New infections
- New Recovery
- Newly Susceptible
- Immunity loss Delay
- New Illness
- Prevalence
- Recovery Delay
- Initial Population
- New Recovery
Classic Feedbacks

Susceptibles

Contacts of Susceptibles with Infectives

Infectives

New Infections
Dynamics

State variables over time

S : Alternative 30 HC Workers Exogenous Recovery Delay
I : Alternative 30 HC Workers Exogenous Recovery Delay
R : Alternative 30 HC Workers Exogenous Recovery Delay
Broadening the Model Boundaries: Endogenous Recovery Delay

- Mean Contacts Per Capita
- Per infected contact infection rate
- Initial Population
- Mean Infectious Contacts Per Susceptible
- Per Susceptible Incidence Rate
- Fractional Prevalence
- Total Population
- Staff Time per Patient
- Healthcare Workers
- Time Until Seek Treatment

Diagram:
- S: Susceptible
- I: Infectious
- R: Recovered

- New infections
- New Recovery
- Newly Susceptible
- Immunity loss Delay
- Recovery Delay
- Healthcare Workers
- Staff Time per Patient
- Time Until Seek Treatment
Broadening the Model Boundaries: Endogenous Recovery Delay

Susceptibles

Contacts of Susceptibles with Infectives

Infectives

New Infections

People Presenting for Treatment

Waiting Times

Health Care Staff

Susceptibles

+ + +

Infectives

New Infections

People Presenting for Treatment

Waiting Times

Health Care Staff

Susceptibles

+ + +

Infectives

New Infections

People Presenting for Treatment

Waiting Times

Health Care Staff
A Different Behaviour Mode

Prevalence, Infectious

Time (Day)

Prevalence : Baseline 30 HC Workers
I : Baseline 30 HC Workers

Person
Structure as Shaping Behaviour

• System structure is defined by
  – Stocks
  – Flows
  – Connections between them

• Nonlinearity: The behaviour of the whole is more than the sum of the behaviour of the parts
  – “Emergent” behaviour would not be anticipated from simple behaviour of each piece in turn

• Stock and flow structure (including feedbacks) of a system determines the qualitative behaviour modes that the system can take on