Introduction to Stocks & Flows

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CMPT 858

State of the System: Stocks (“Levels”, “State Variables”, “Compartments”)

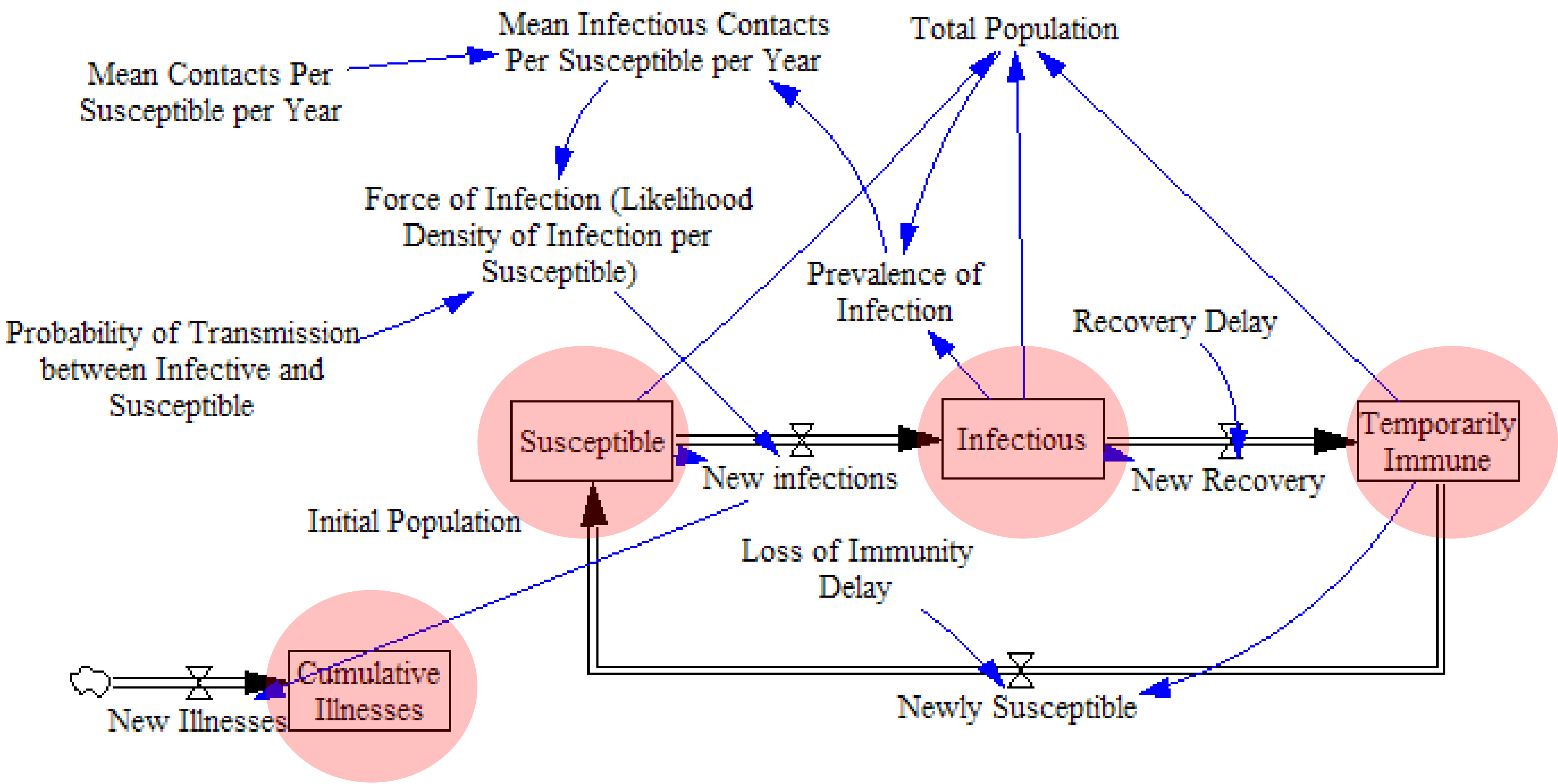
- Stocks (Levels) represent accumulations
  - These capture the “state of the system”
  - Mathematically, we will call these “state variables”
- These can be measured at *one instant in time*
- Stocks start with some initial value & are thereafter changed only by *flows* into & out of them
  - There are no inputs that immediately change stocks
- Stocks are the source of delay in a system
- In a stock & flow diagram, shown as ***rectangles***

Examples of Stocks

- Water in a tub or reservoir
- People of different types
  - {Susceptible, infective, immune} people
  - Pregnant women
  - Women between the age of x and *y*
  - High-risk individuals
- Healthcare workers
- Medicine in stocks

- Money in bank account
- CO_2 in atmosphere
- Blood sugar
- Stored Energy
- Degree of belief in X
- Stockpiled vaccines
- Goods in a warehouse
- Beds in an emergency room
- Owned vehicles

Example Model: Stocks



The Critical Role of Stocks in Dynamics

- Stocks determine current state of system
  - Stocks often provide the basis for making choices
- Stocks central to most disequilibria phenomena (buildup, decay)
- Lead to inertia
- Give rise to delays

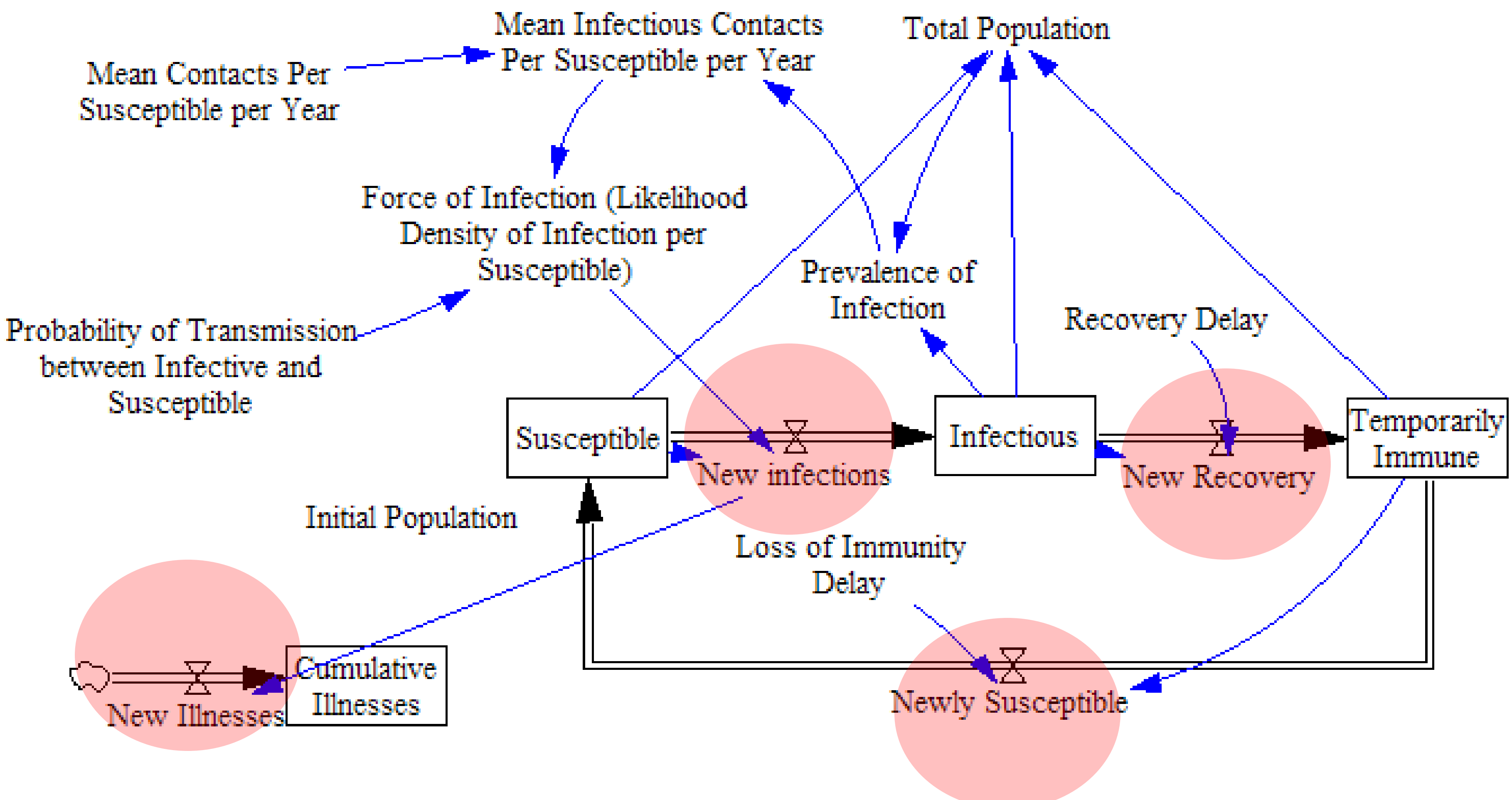
State Changes: Flows ("Fluxes", "Rates", "Derivatives")

- All changes to stocks occur via *flows*
- Always expressed per some unit time: If these flow into/out of a stock that keeps track of things of type *X (*e.g. persons), the rates are measured in *X*/(Time Unit) (e.g. persons/year, $/month, gallons/second)
- Typically measure over certain period of time (by considering accumulated quantity over a period of time)
  - e.g. Incidence Rates is calculated by accumulating people over a year, revenue is $/Time, water flow is litres/minute
  - Can be estimated for any point in time

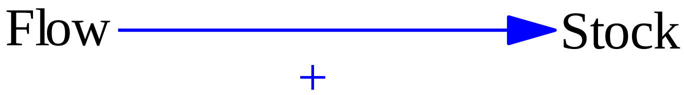
Examples of Flows

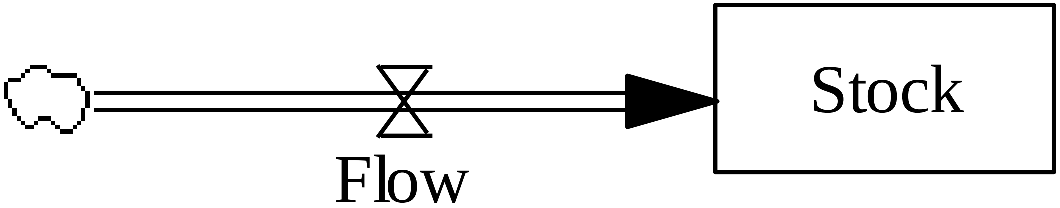
- Inflow or outflow of a bathtub (litres/minute)
- Rate of incident cases (e.g. people/month)
- Rate of recovery
- Rate of mortality (e.g. people/year)
- Rate of births (e.g. babies/year)
- Rate of treatment (people/day)
- Rate of caloric consumption (kcal/day)
- Rate of pregnancies (pregnancies/month)
- Reactivation Rate (# of TB cases reactivating per unit time)
- Revenue ($/month)
- Spending rate ($/month)
- Power (Watts)
- Rate of energy expenditure
- Vehicle sales
- Vaccine sales
- Shipping rate of goods

Example Model: Flows

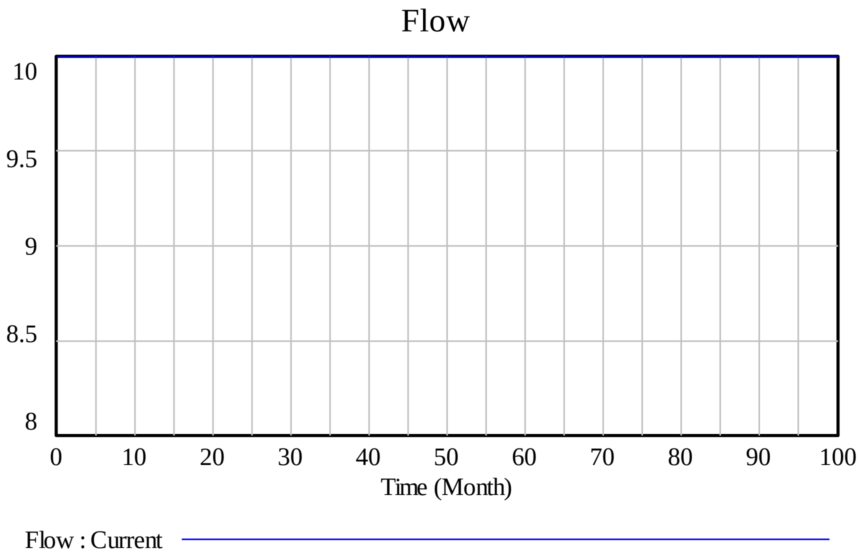


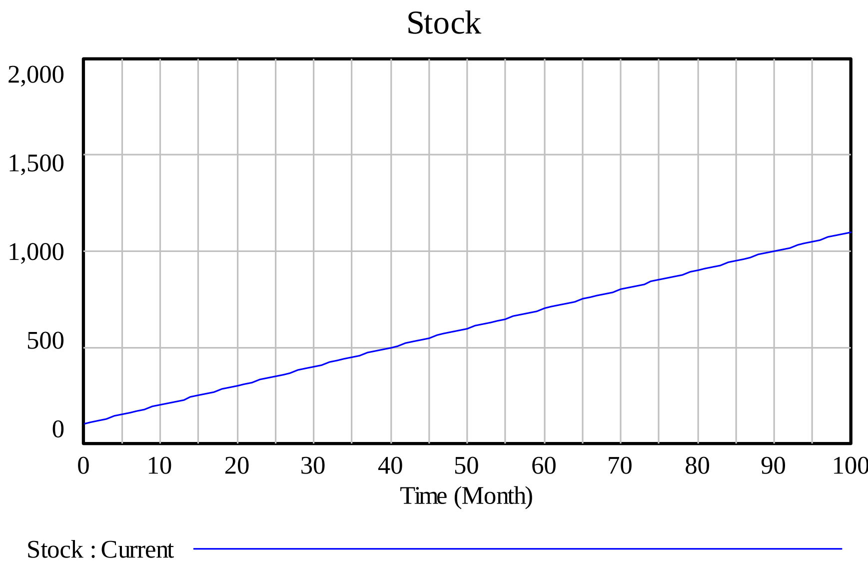
Key Component: Stock & Flow



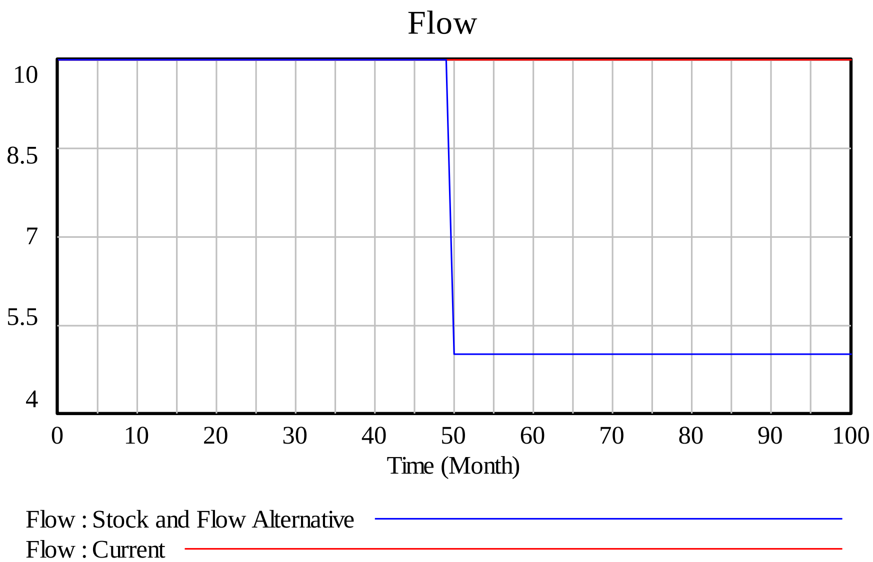


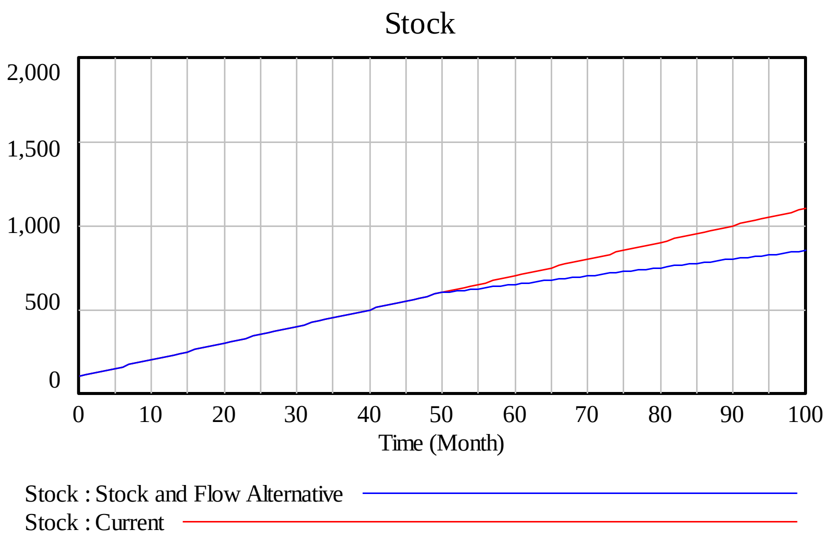
Net Flow Impact on Stock





Impact of Lowering Flow (Rate) to 5/Month?





Loops & Stocks

- Causation does not effect big change instantaneously
  - Loops are not instantaneous
- Stocks only change by changes to the flows into & out of them
  - There are no inputs that immediately change stocks
- All causal loops must involve at least one stock
  - The state of the world must change as part of the process
  - Absent a stock, loop would be instantaneous

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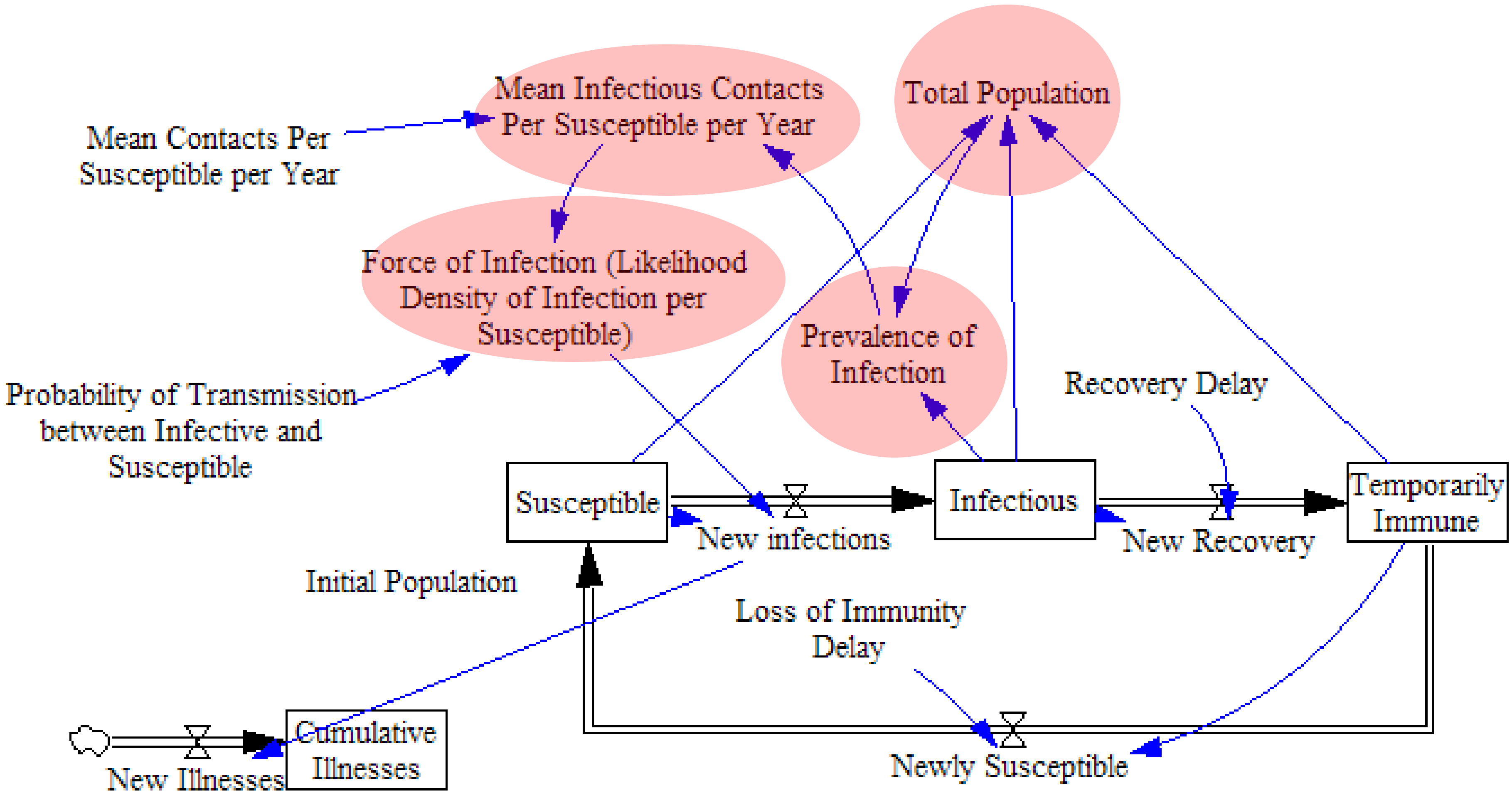
Auxiliary Variables

- Auxiliary variables are convenience names we give to concepts that can be defined in terms of expressions involving stocks/flows at current time
  - Adding or eliminating an auxiliary variable does not change the mathematical structure of the system
- Critical for model transparency
  - Can be reused at many places
  - References to auxiliary variables prevents need for modeler to think about all of details of definition
- Enhanced modifiability: Single place to define
- Convenient for reporting (graphing, tables) & analyzing model dynamics

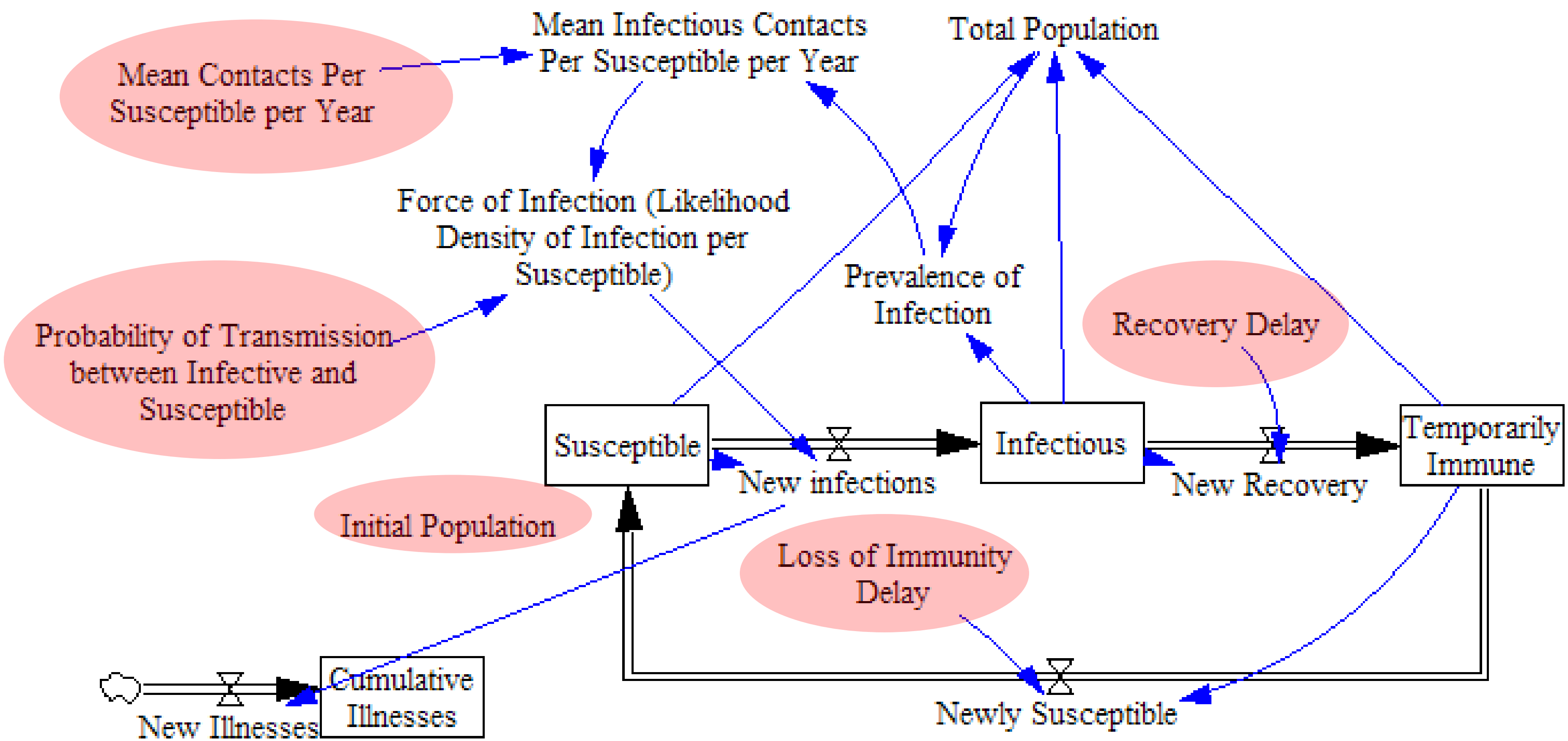
Example Model: Auxiliary Variables



Constants & Time Series Parameters

- For similar reasons to auxiliary variables, we give names to
  - Model constants
  - Time series

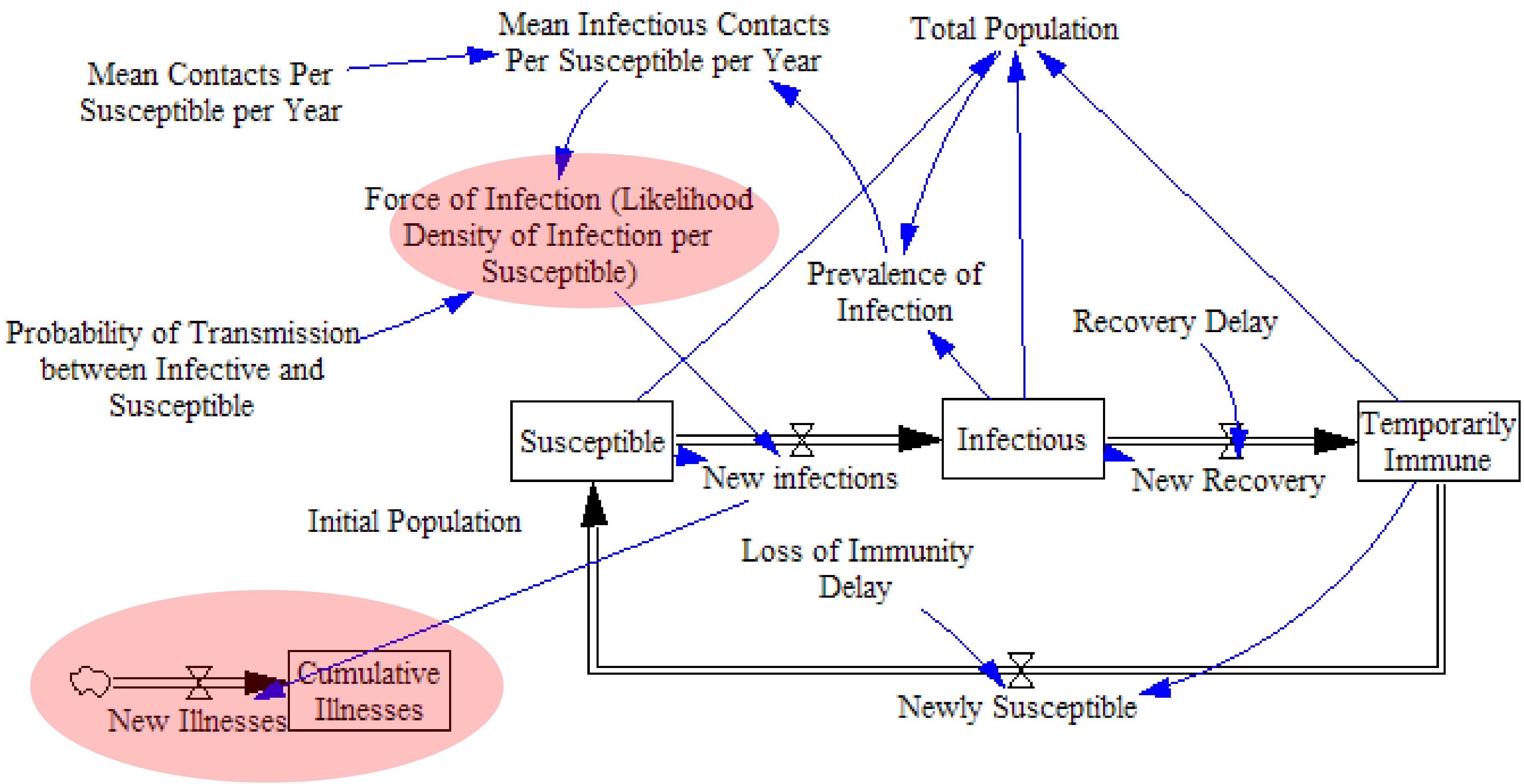
Example Model: Parameters



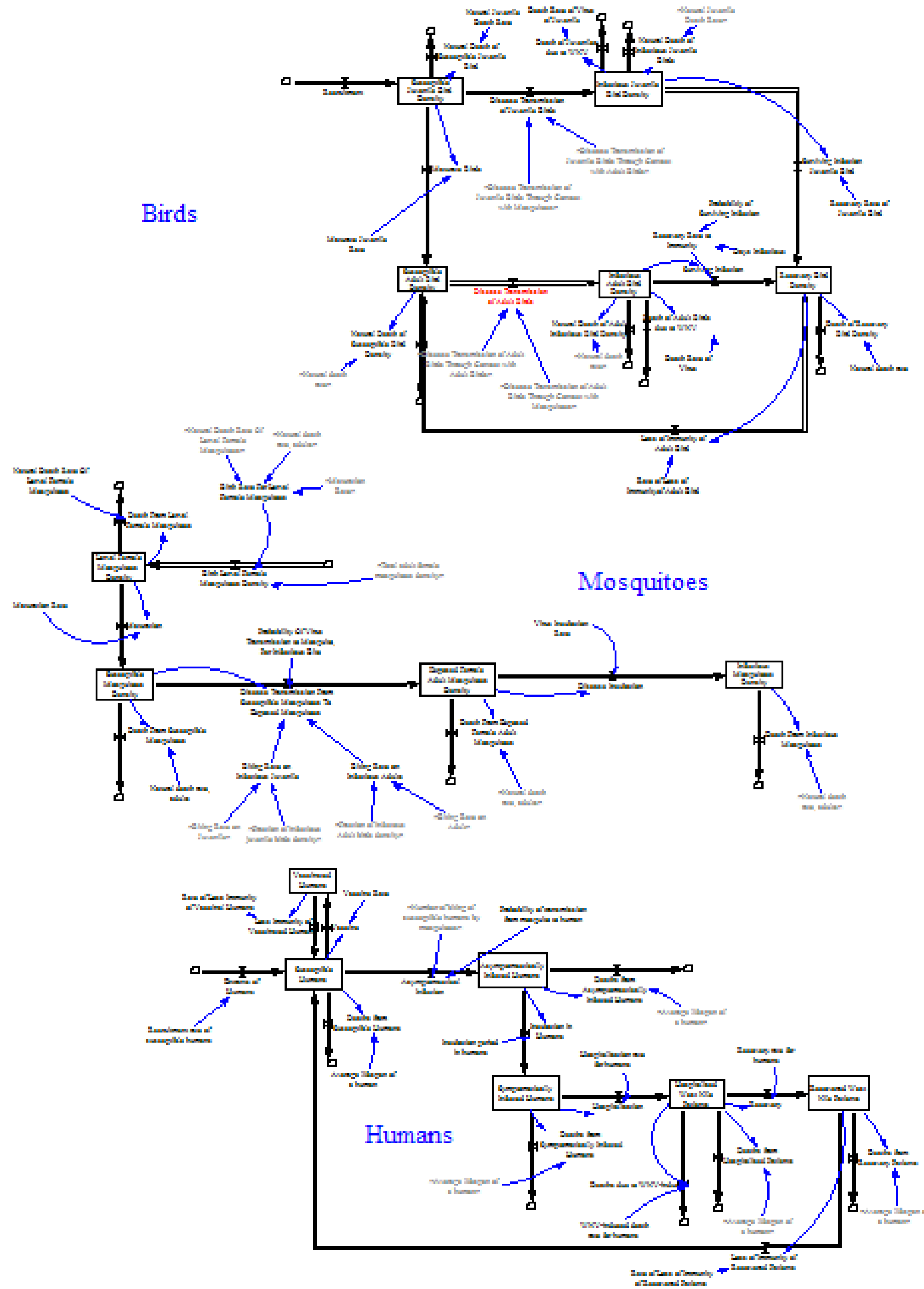
Stocks & Flows Compared with Markov Models

- Open population
  - Births
  - Deaths
- Non-constant likelihood (density) of transitions
  - Likelihood of leaving a stock per unit time can depend on other stocks
    - Force of Infection (likelihood of susceptible becoming infected) can depend on prevalence of illness
    - Likelihood of initiating smoking could depend on accumulated current or former smokers
- Multiple types of stocks
  - e.g. costs, QALYs, hosts & reservoir species, etc.
- Continuous time

Distinctive Stock & Flow Features



Multi-Species Model (West Nile Virus)

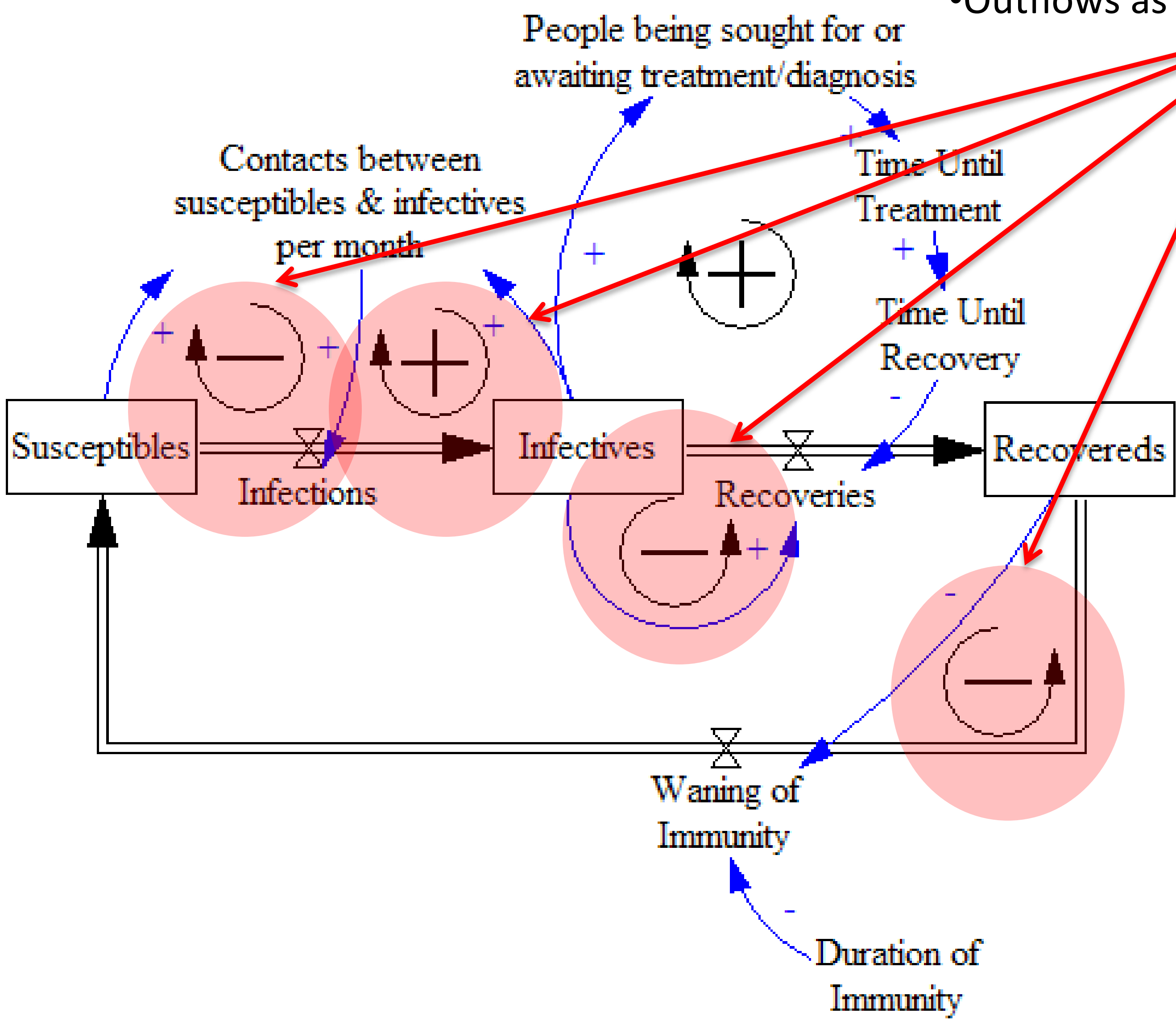


Refinement of Causal Loop Diagrams: System Structure Diagrams

- Still essentially a qualitative model, but less ambiguous
  - By clearly distinguish stocks & flows, this helps reduce the artifactual loops discussed with CLDs
- Combine causal loops diagram elements with stock & flow structure
- If complete, all loops will go “through a stock”
  - Loop goes into the flow of a stock (as one variable in the diagram)
  - Loop comes comes out of stock (as next variable in diagram)

Example System Structure Diagram

Note treatment of flows as links from flow to stock
•Inflows as positive links
•Outflows as negative links



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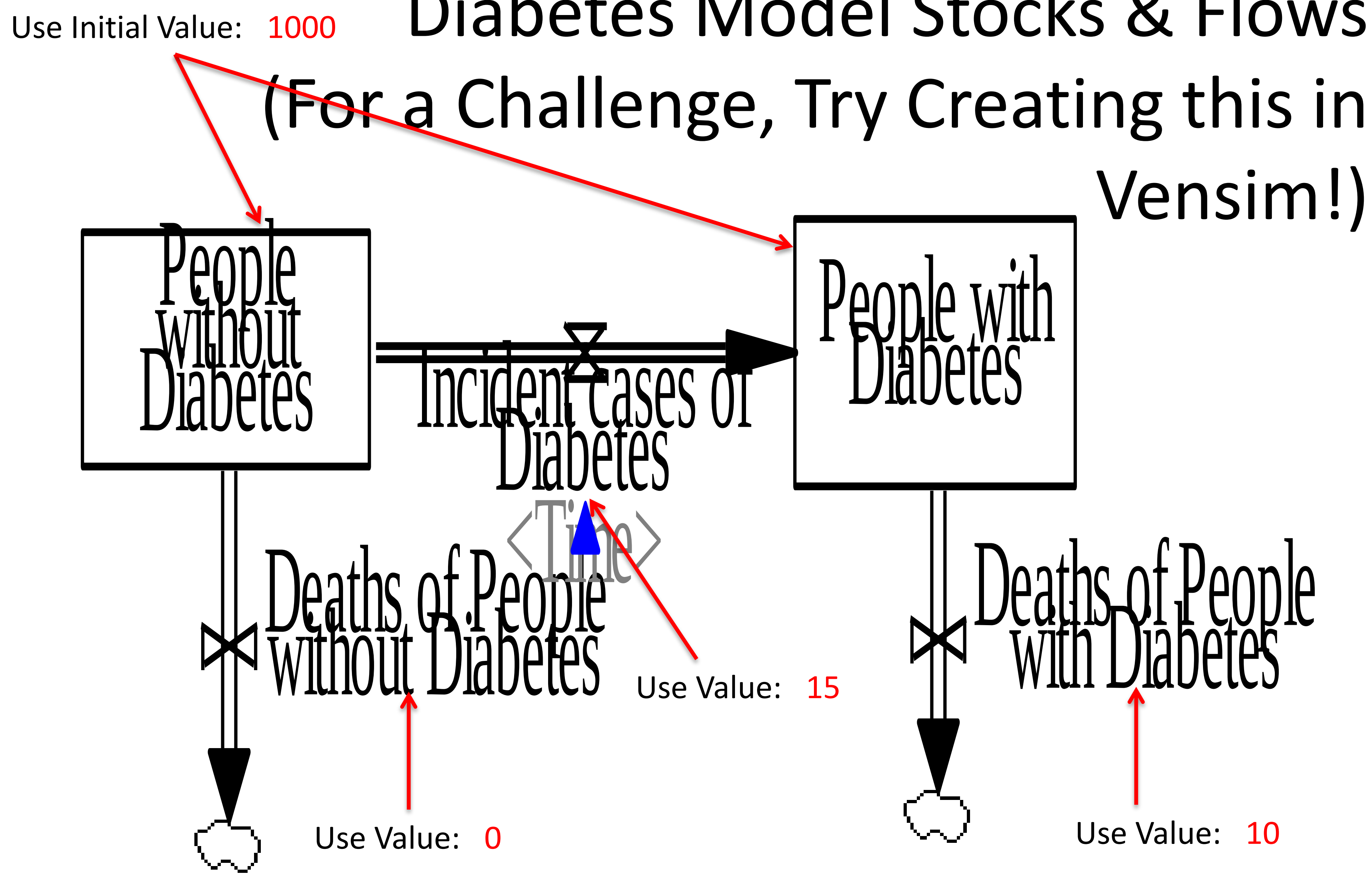
Stocks & Flows: Diabetes

- Assume diabetes is not curable
- Stocks:
  - People without diabetes (at different stages of risk?)
  - People with diabetes
- Flows
  - Incident cases (both diagnosed & undiagnosed!)
  - Deaths from both stocks

Stocks & Flows: Tuberculosis

- Assume that TB infection cannot be totally eliminated
- Stocks
  - Susceptible people
  - Immunized people
  - People with latent TB infection
  - People with active TB infection
- Flows
  - People becoming latently infected
  - People being vaccinated
  - People with infection going to Active TB ("primary progression"
  - People with infection going on to latent TB
  - People with secondary infection going on to active TB
  - Deaths from each stock

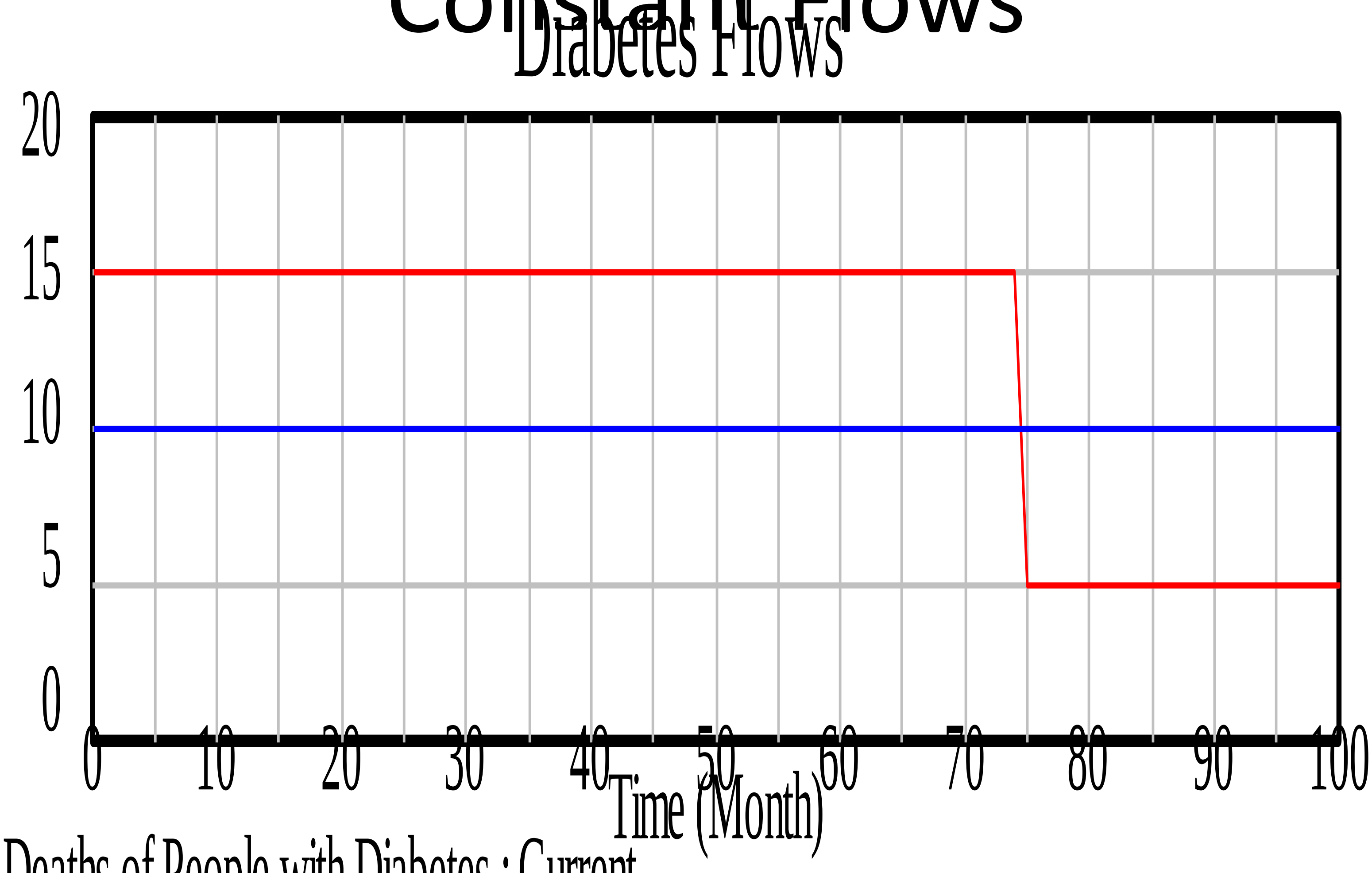
Diabetes Model Stocks & Flows (For a Challenge, Try Creating this in Vensim!)



Interactive Steps

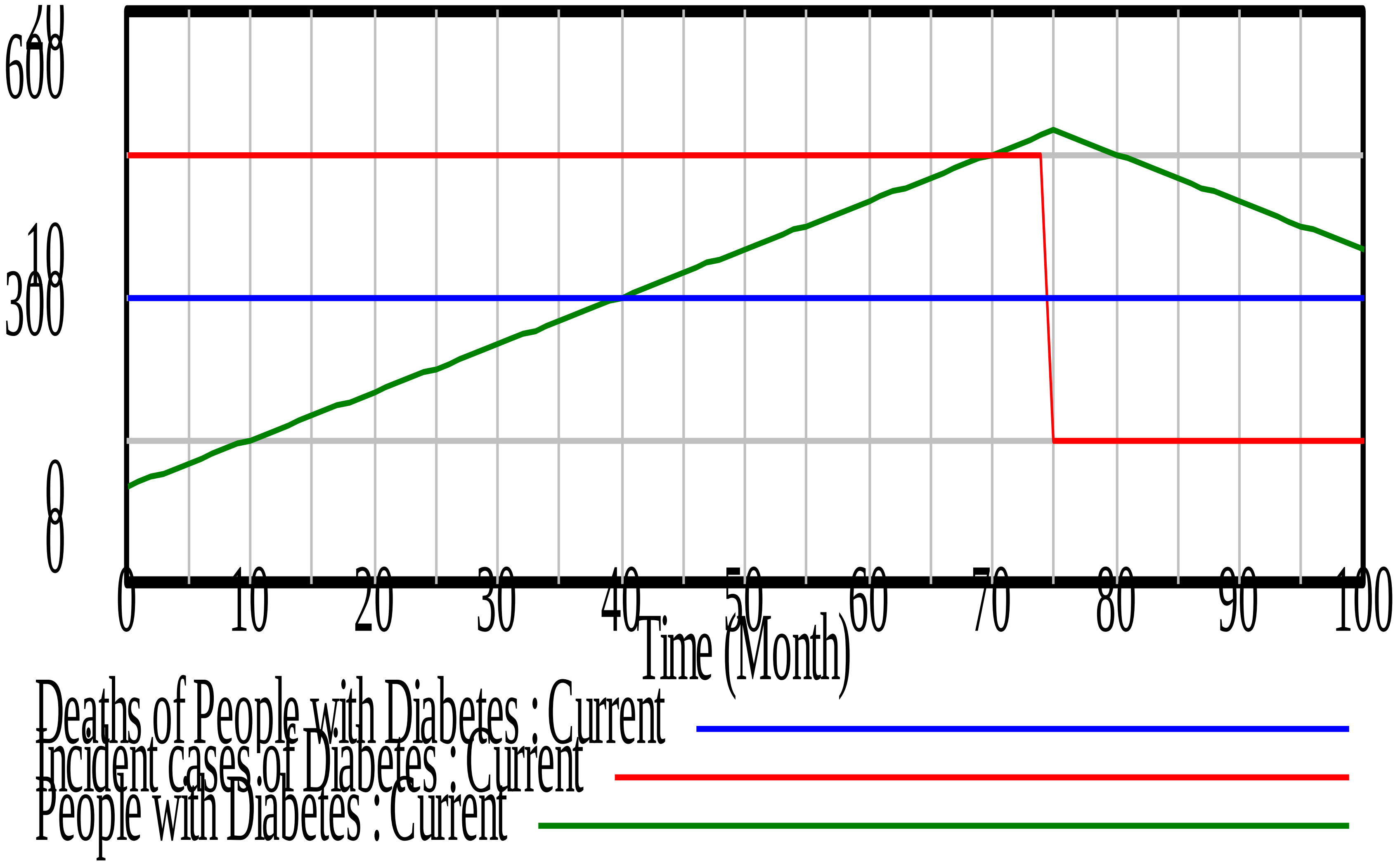
- View flows and stocks – does this make sense?
- Hitch up constant “auxiliary” variables to flows
- How does changing constant variables change the stock?

Constant Flows

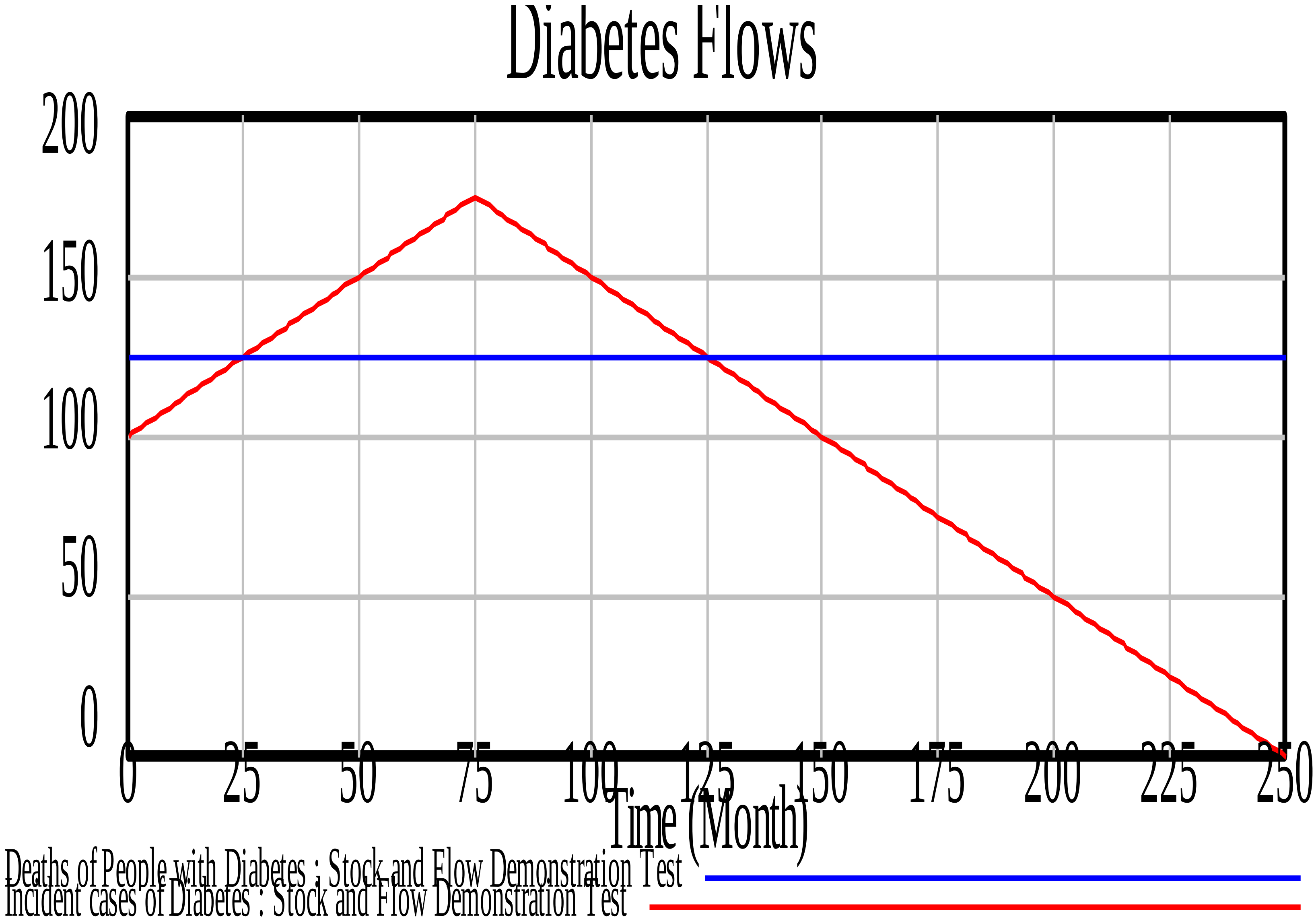


What happens to the stock?

Resulting Stock (Green)

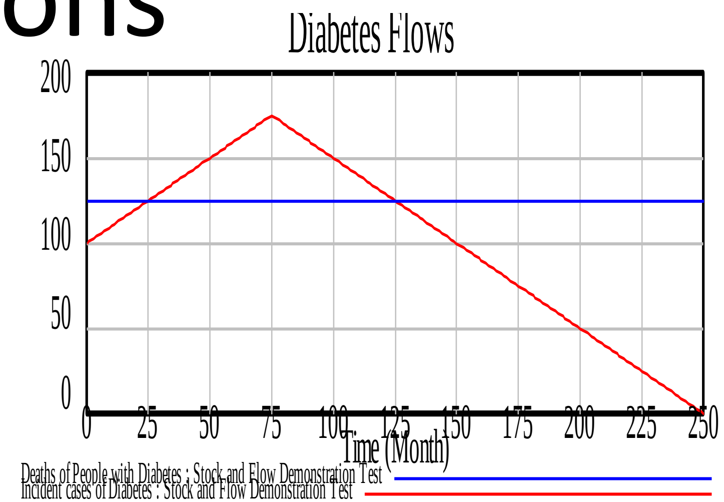


Suppose we have these Flows (Rates)



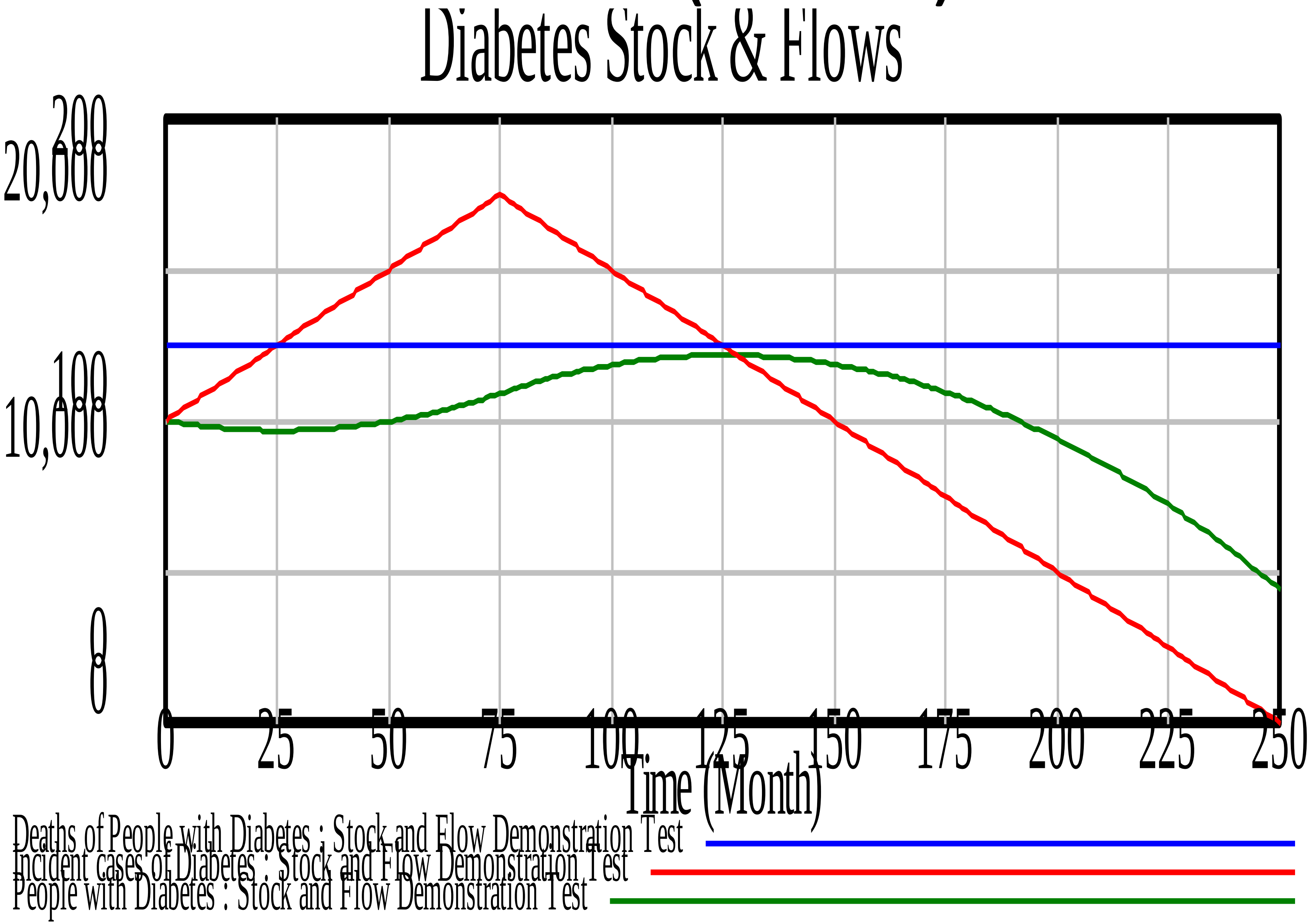
What happens to the stock?

Some Questions



- When is the stock of people with diabetes at its lowest value?
- When is the stock of people with diabetes at its greatest value?
- Is the value of the stock of people with diabetes larger at the beginning or end?
- When is the stock of people with diabetes not changing?

Stock (Green)

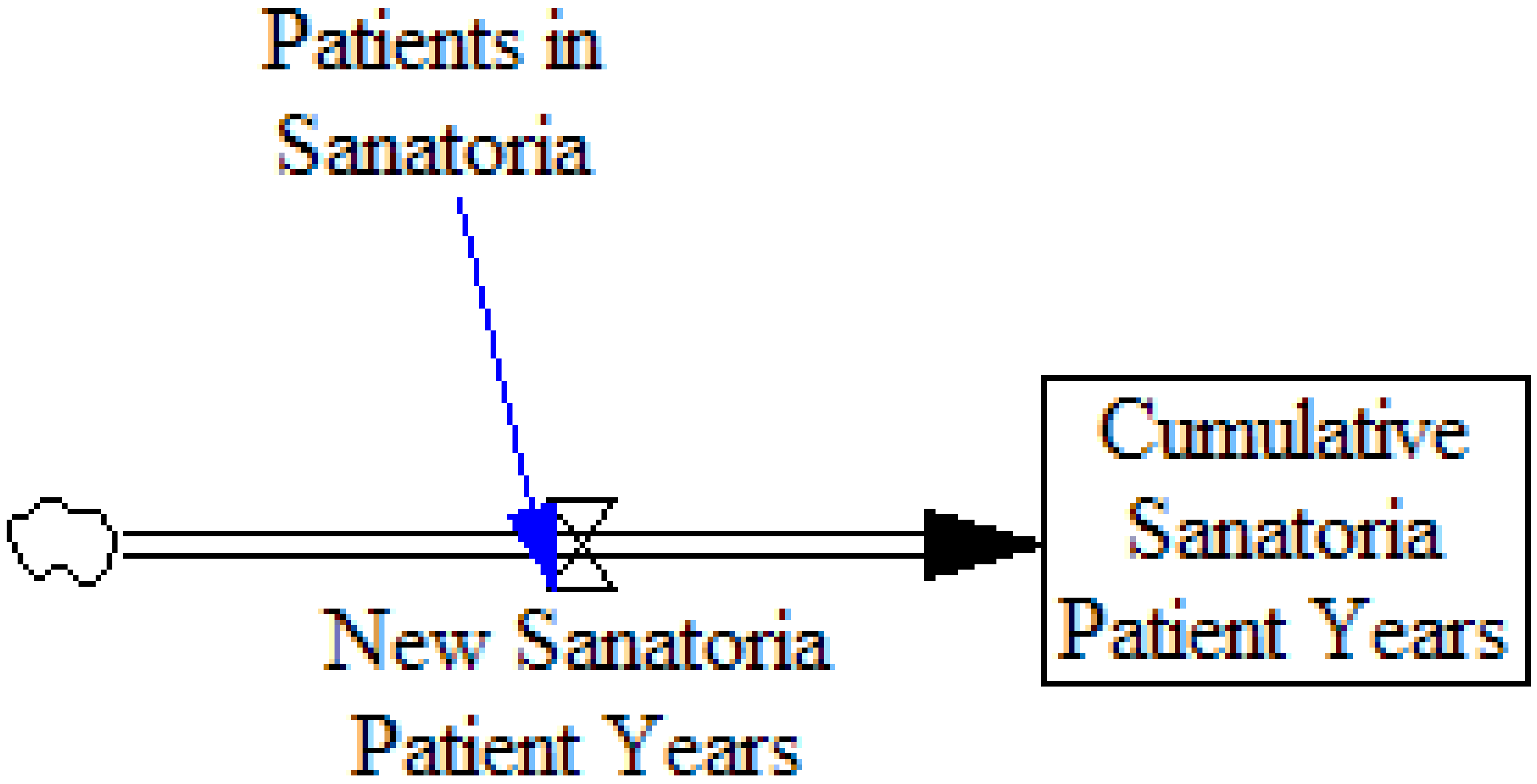


Diabetes Stock & Flows
200
20,000
100
10,000
0
0
0
25
50
75
100
125
150
175
200
225
250
Time (Month)
Deaths of People with Diabetes : Stock and Flow Demonstration Test
Incident cases of Diabetes : Stock and Flow Demonstration Test
People with Diabetes : Stock and Flow Demonstration Test

Flows and Feedbacks

- Stocks are always changed by flows
- In your experiments, we’ve used constant values for flows
- In general, the formulas for the flows will depend on things that are changing (state)
  - Ultimately, these things must depend on the things that collectively specify the state – the stocks!

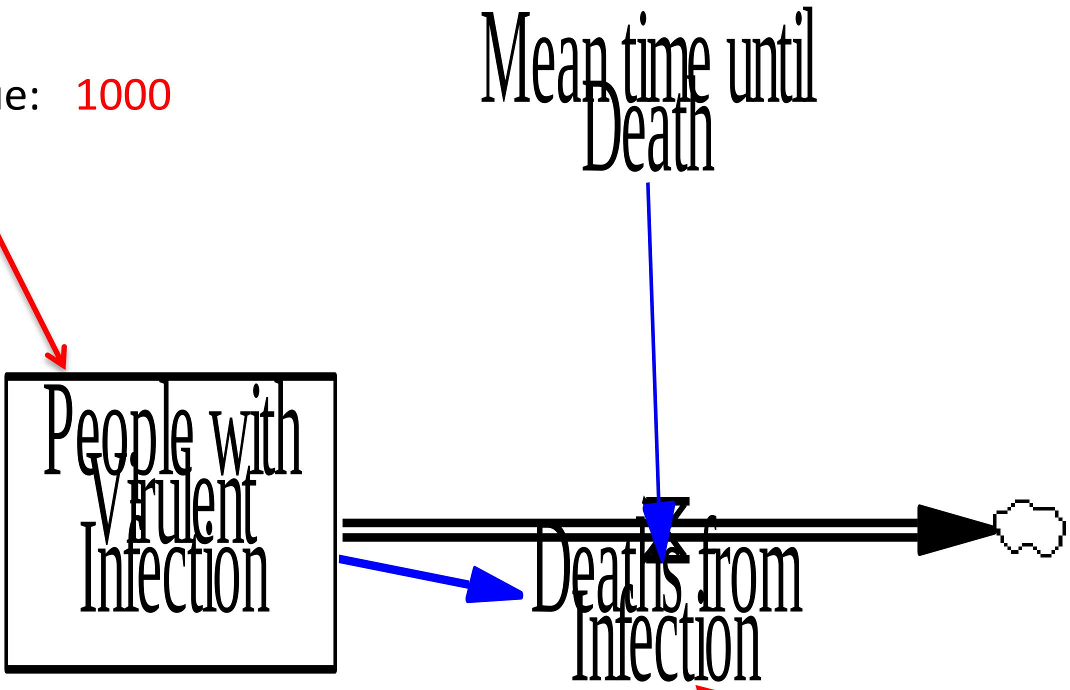
Example 2



Patients in Sanatoria
Cumulative Sanatoria Patient Years
New Sanatoria Patient Years

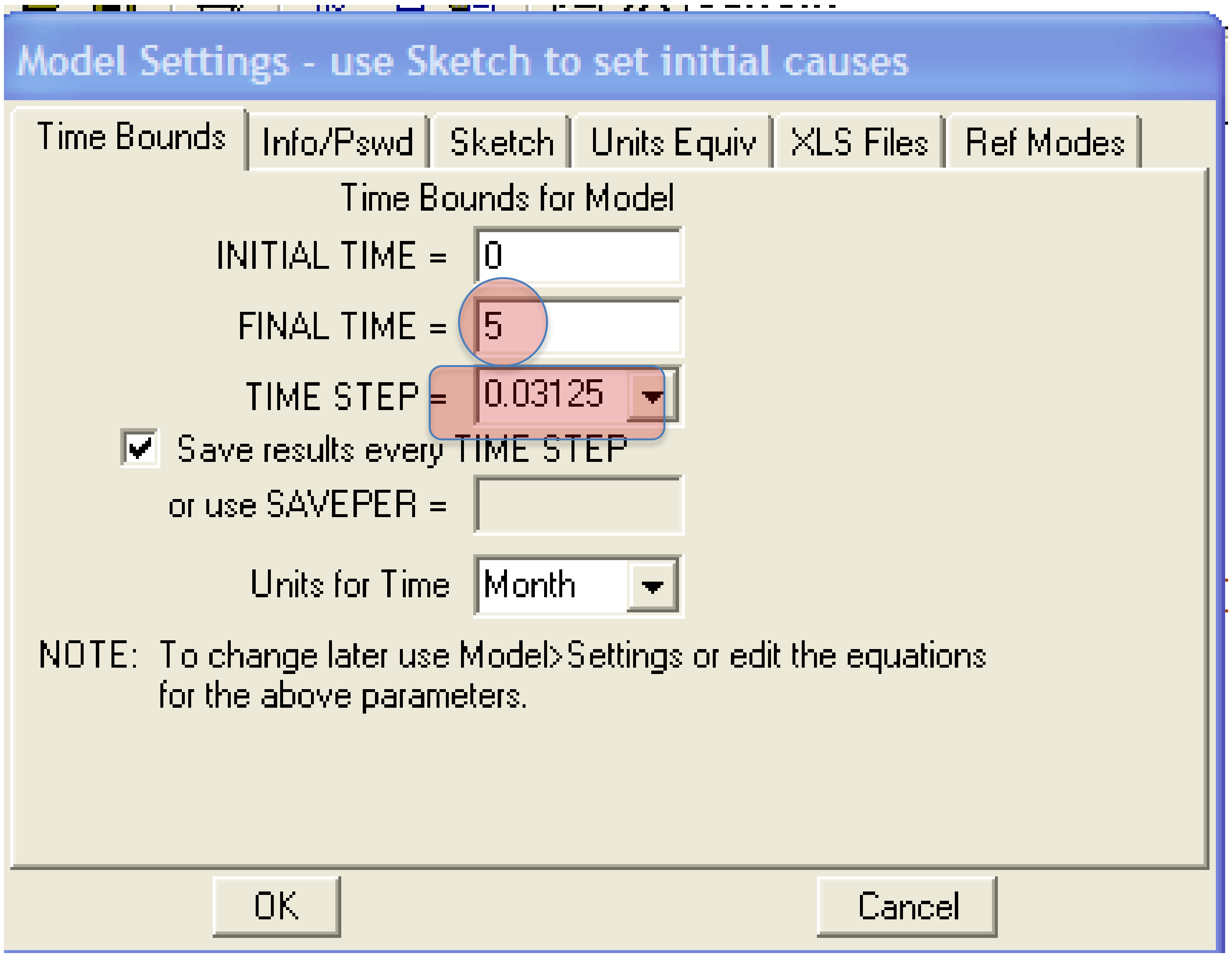
Simple First-Order Decay (Create this in Vensim!)

Use Initial Value: 1000

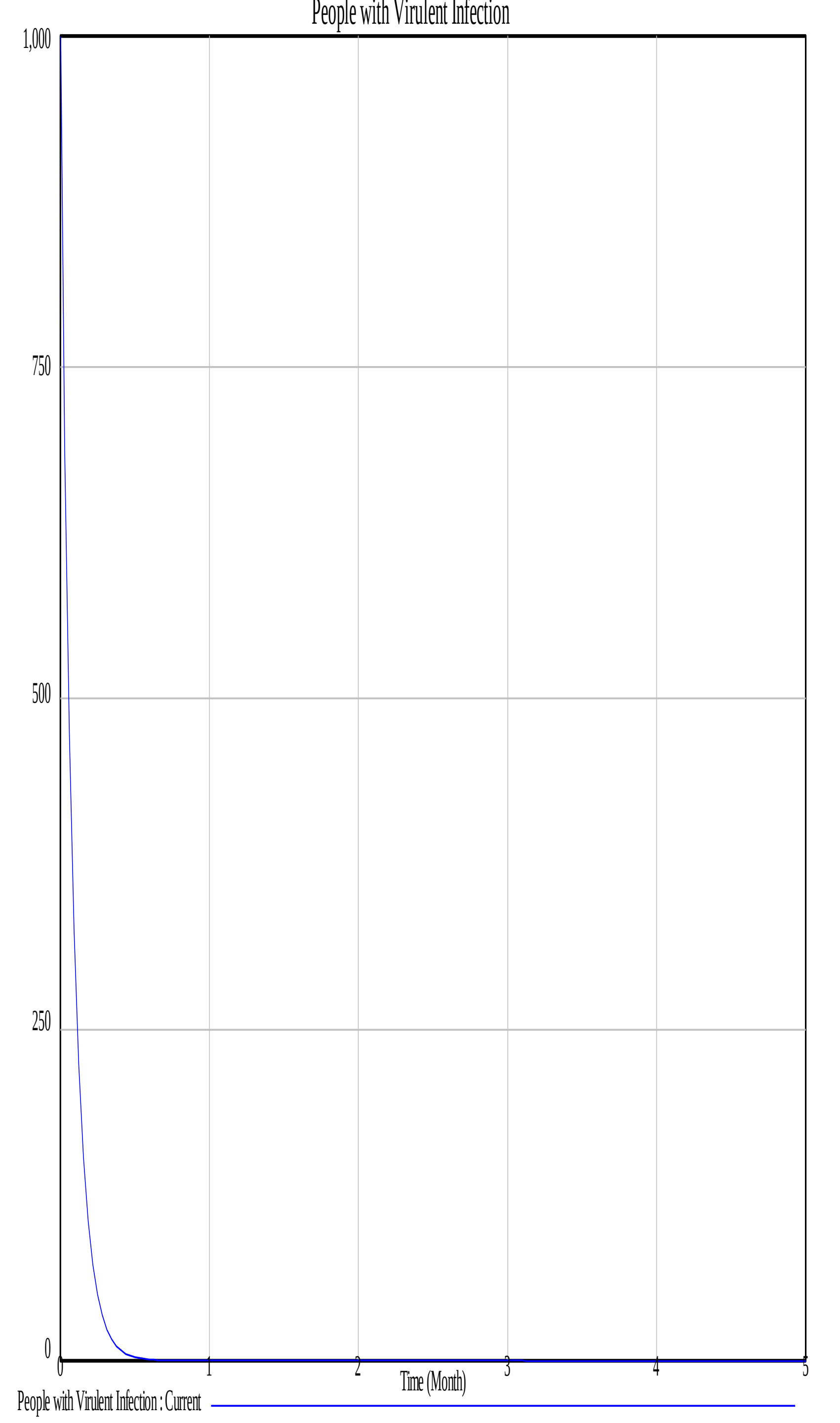


Use Formula: Deaths from Infection/Mean time until Death

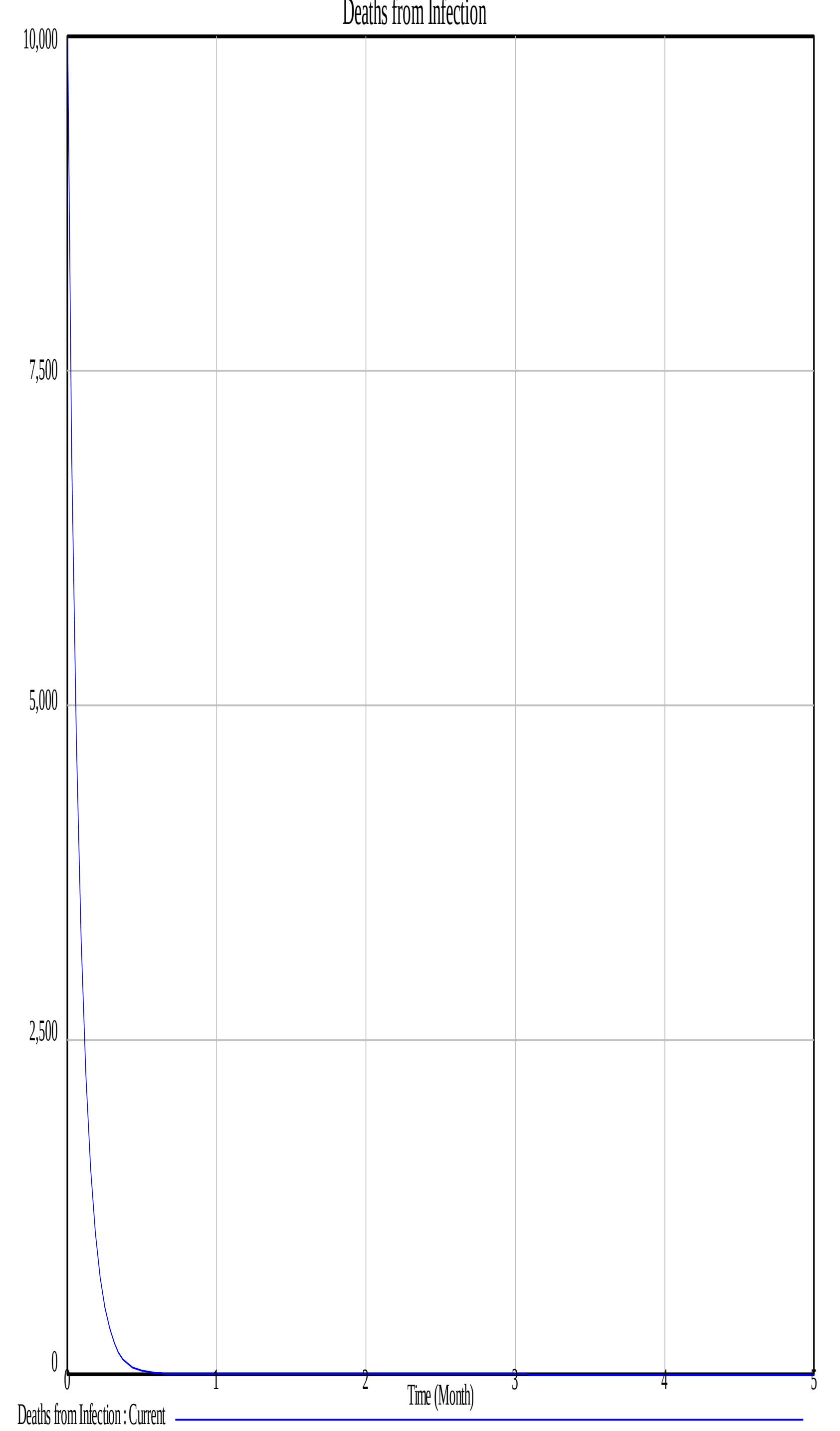
Set Model Settings (Model Menu/Settings Item)



Dynamics of Stock?

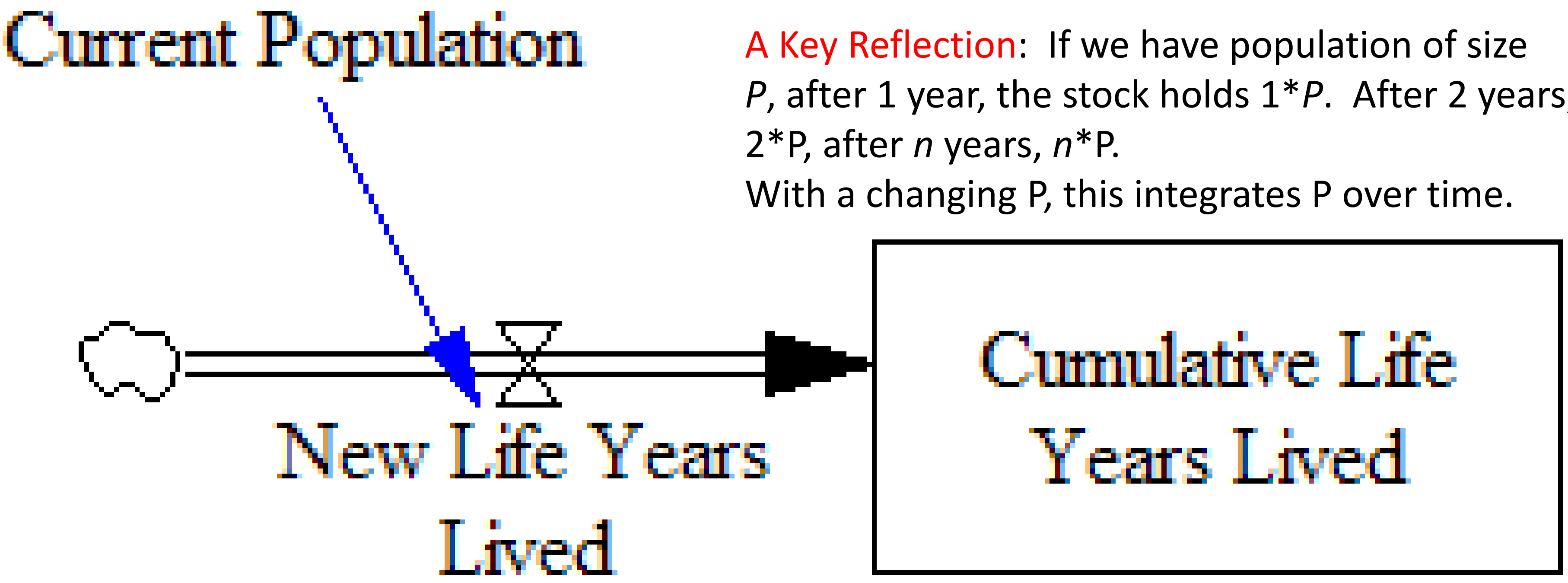


Dynamics of (Rate of) Death Flow?



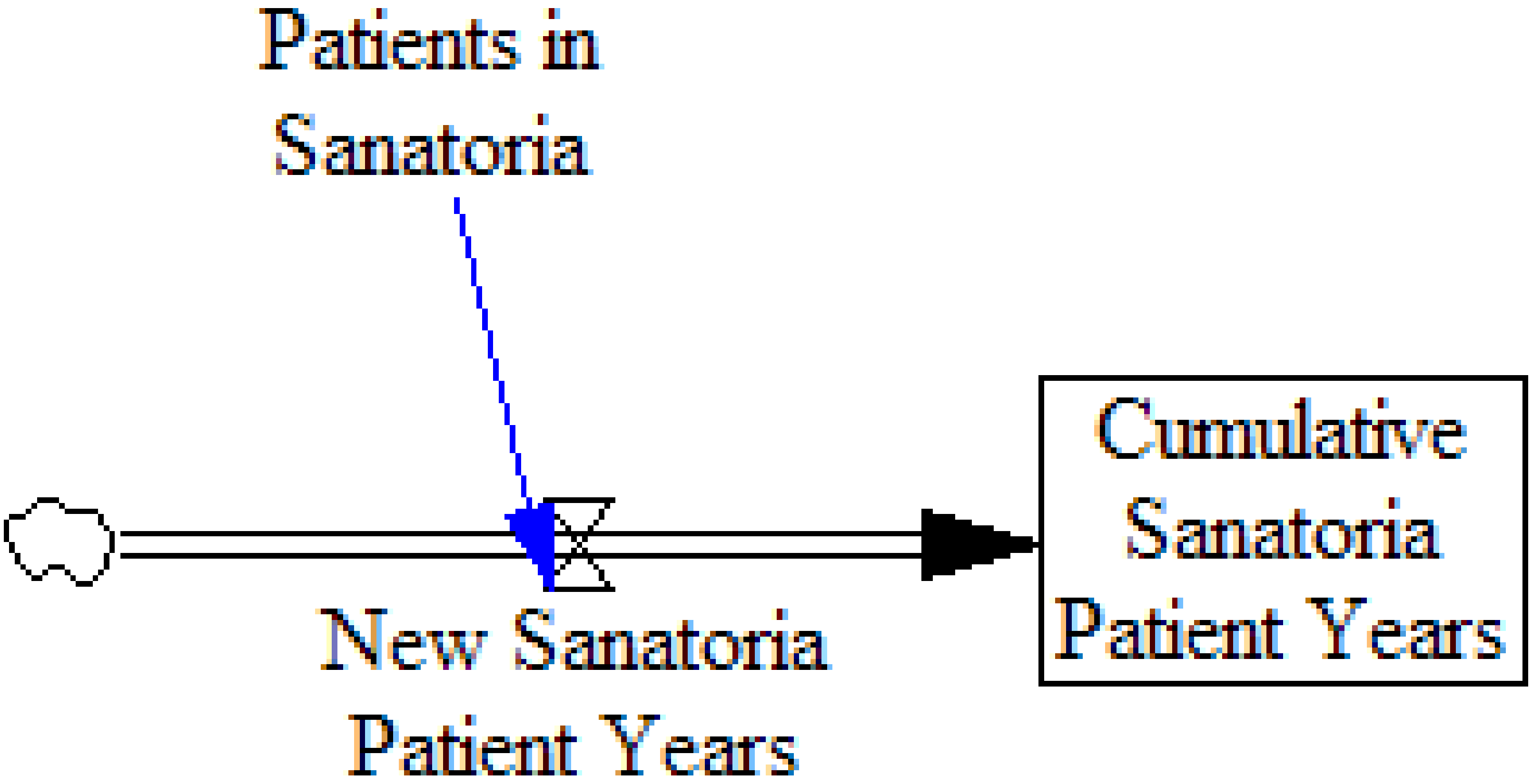
Stocks As Accumulations

- We often use stocks to accumulate (integrate) other (evolving) quantities over time
- Example (assume time measured in years):

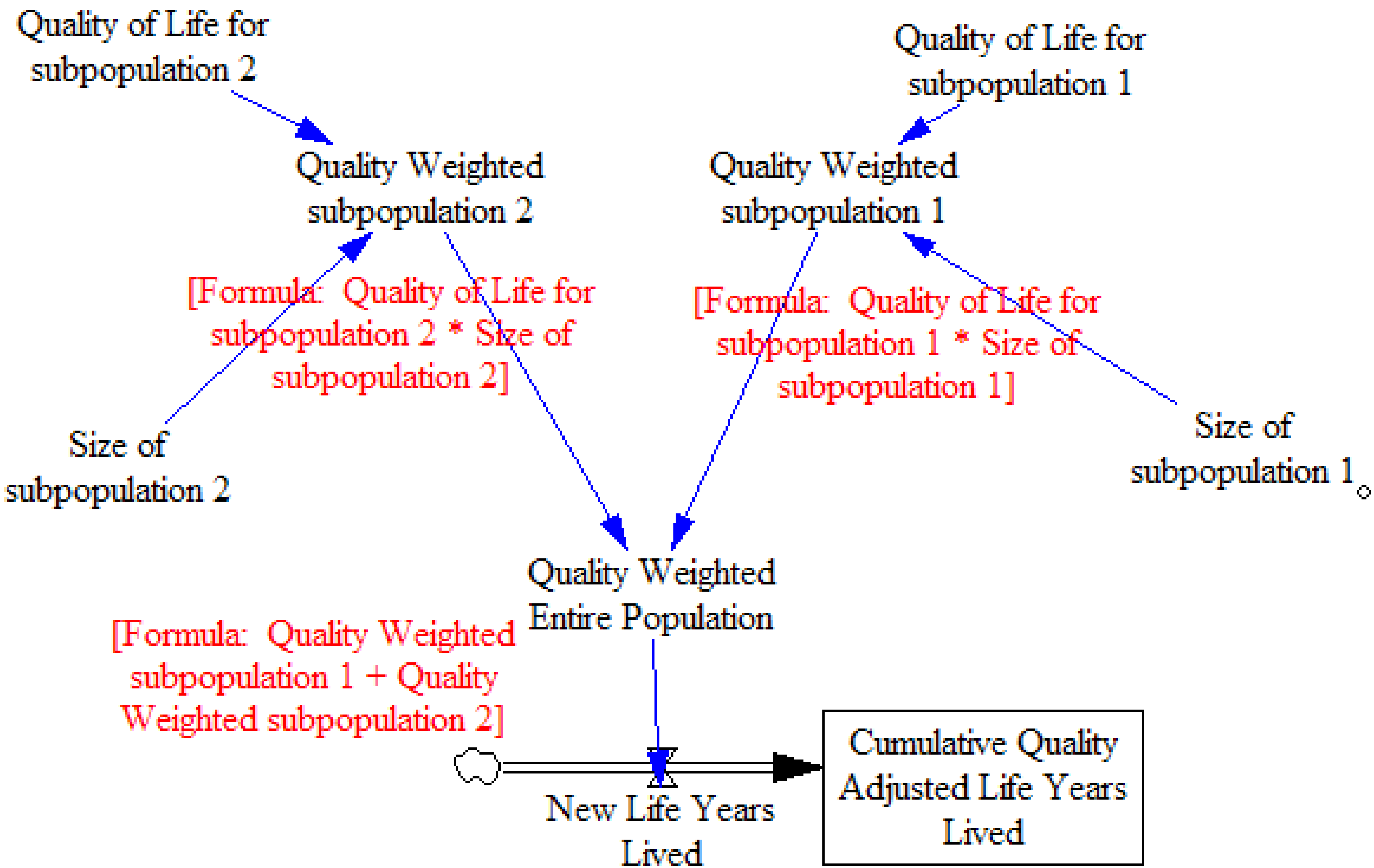


A Key Reflection: If we have population of size *P*, after 1 year, the stock holds 1**P*. After 2 years, 2*P, after *n* years, *n**P.
With a changing P, this integrates P over time.

Example 2


Patients in Sanatoria
New Sanatoria Patient Years
Cumulative Sanatoria Patient Years

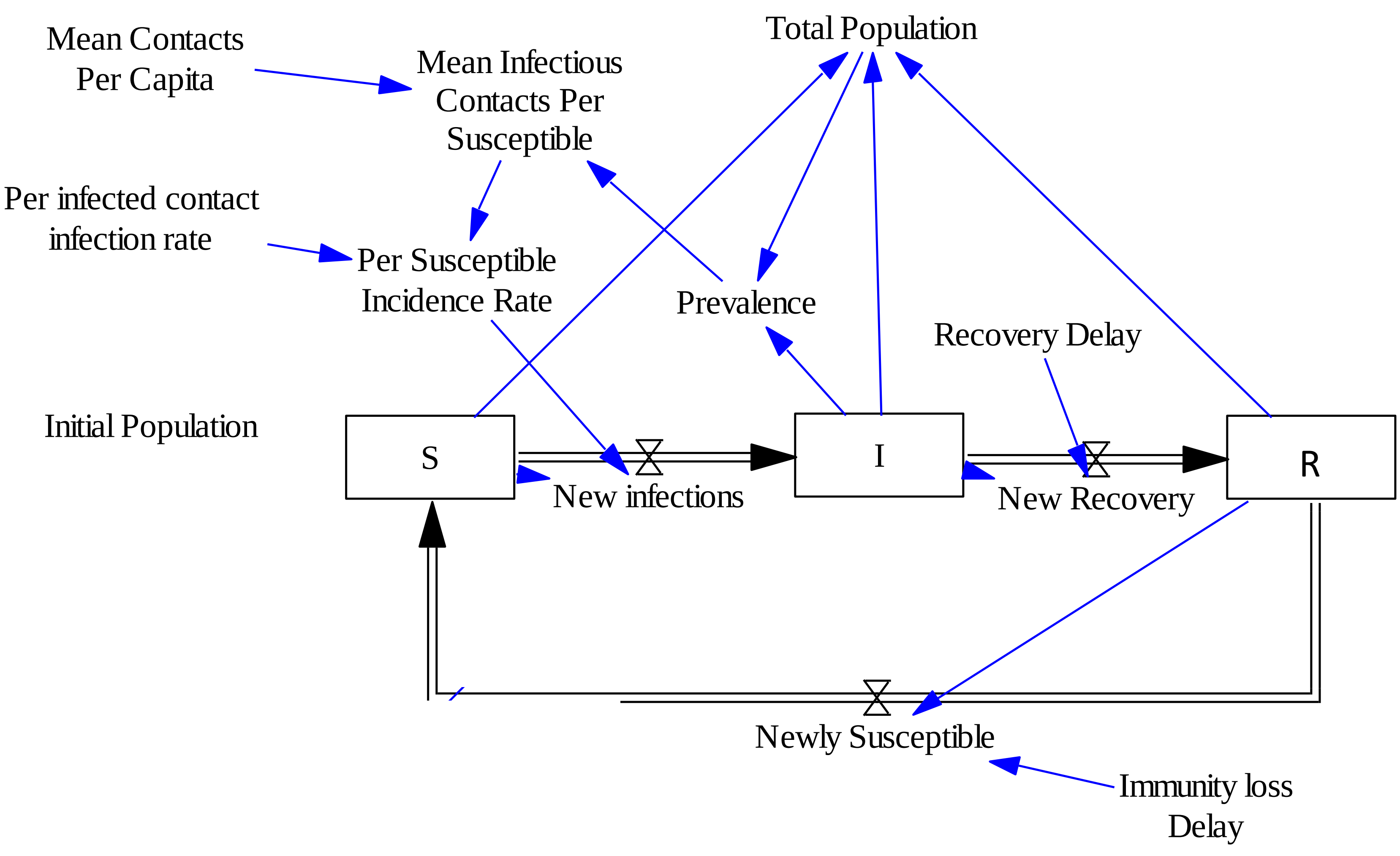
Slightly more Sophisticated



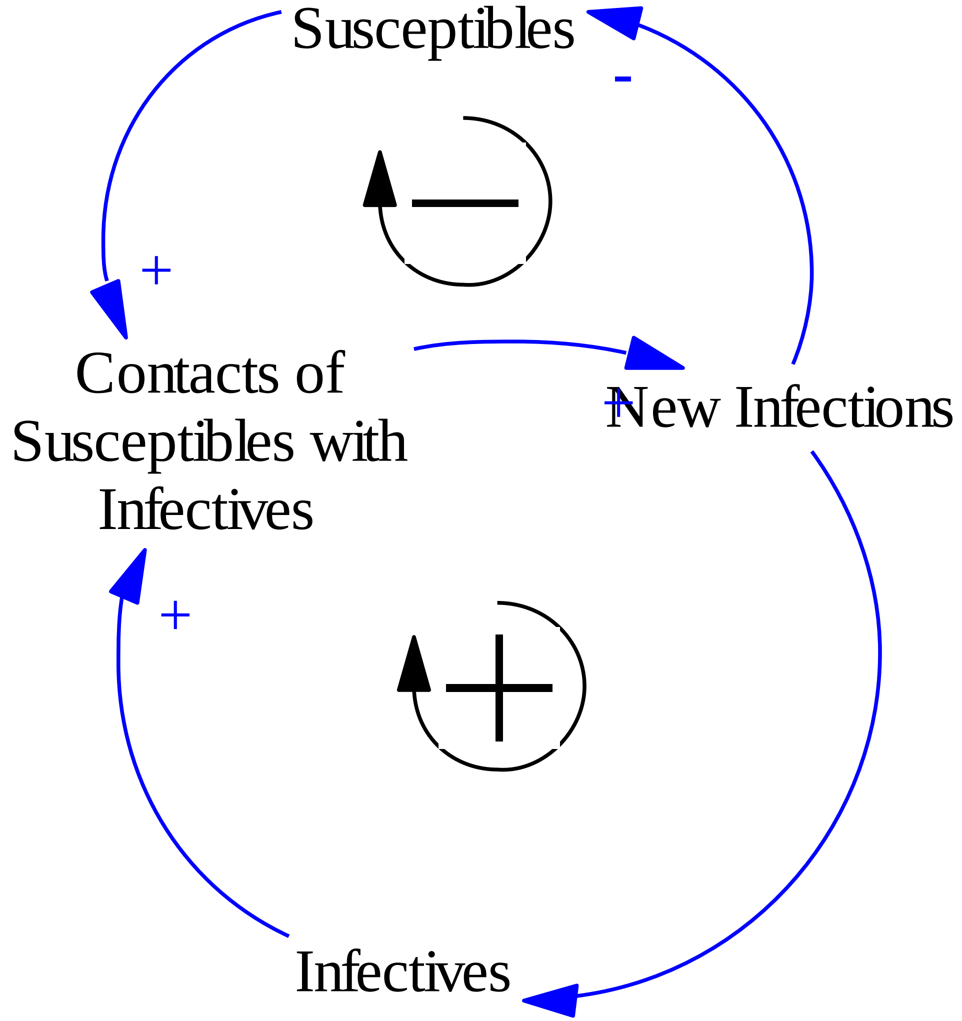
Principle: Structure Determines Behaviour

- Feedback & stock-and-flow structure of a system determines the possible patterns of behaviour
- Different sets of parameters (e.g. values for constants) will select particular behaviour within these behaviour patterns
- Changes to the feedback structure can change behaviour in fundamental ways

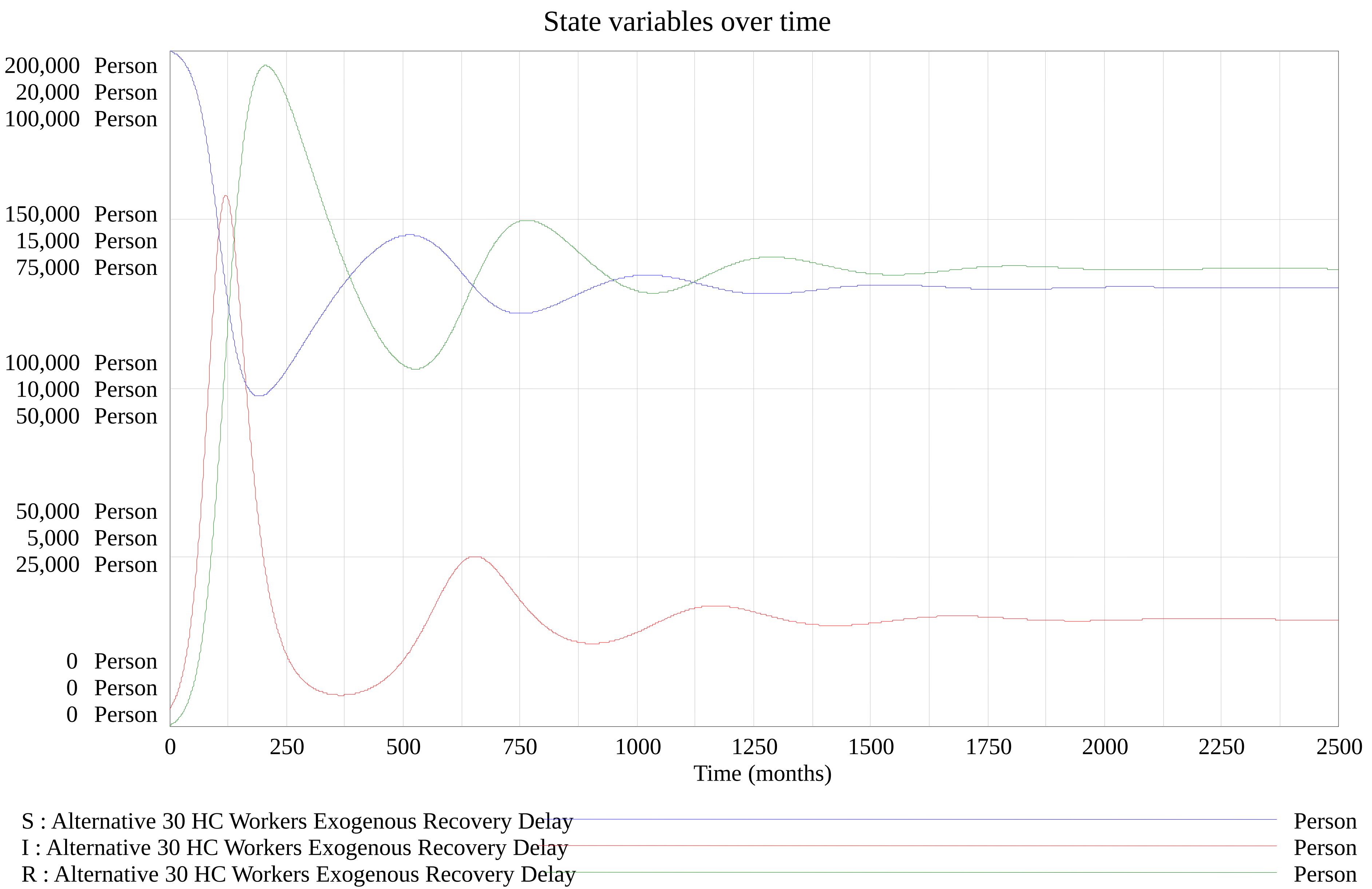
Simple SIT Model



Classic Feedbacks

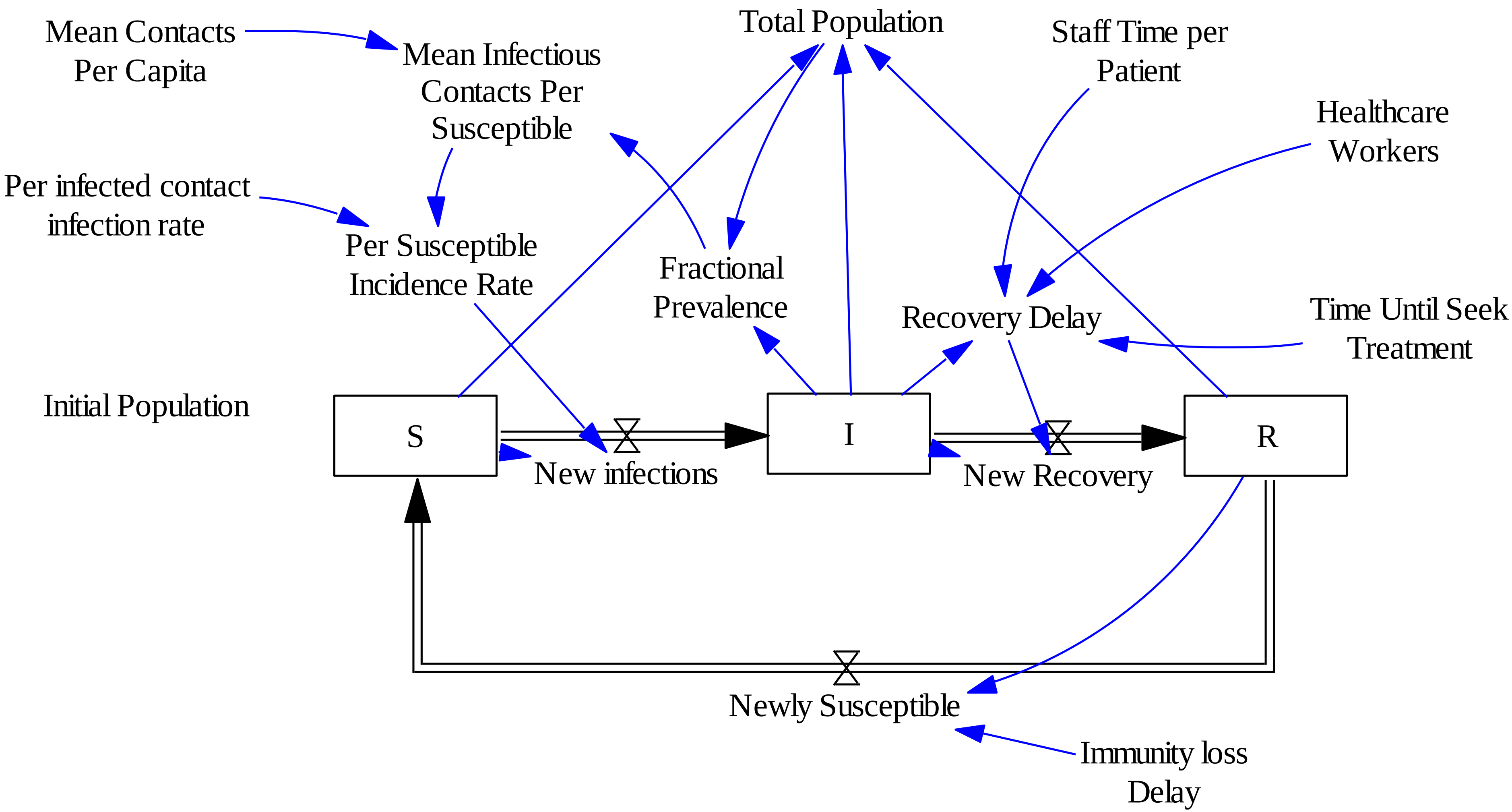


Dynamics

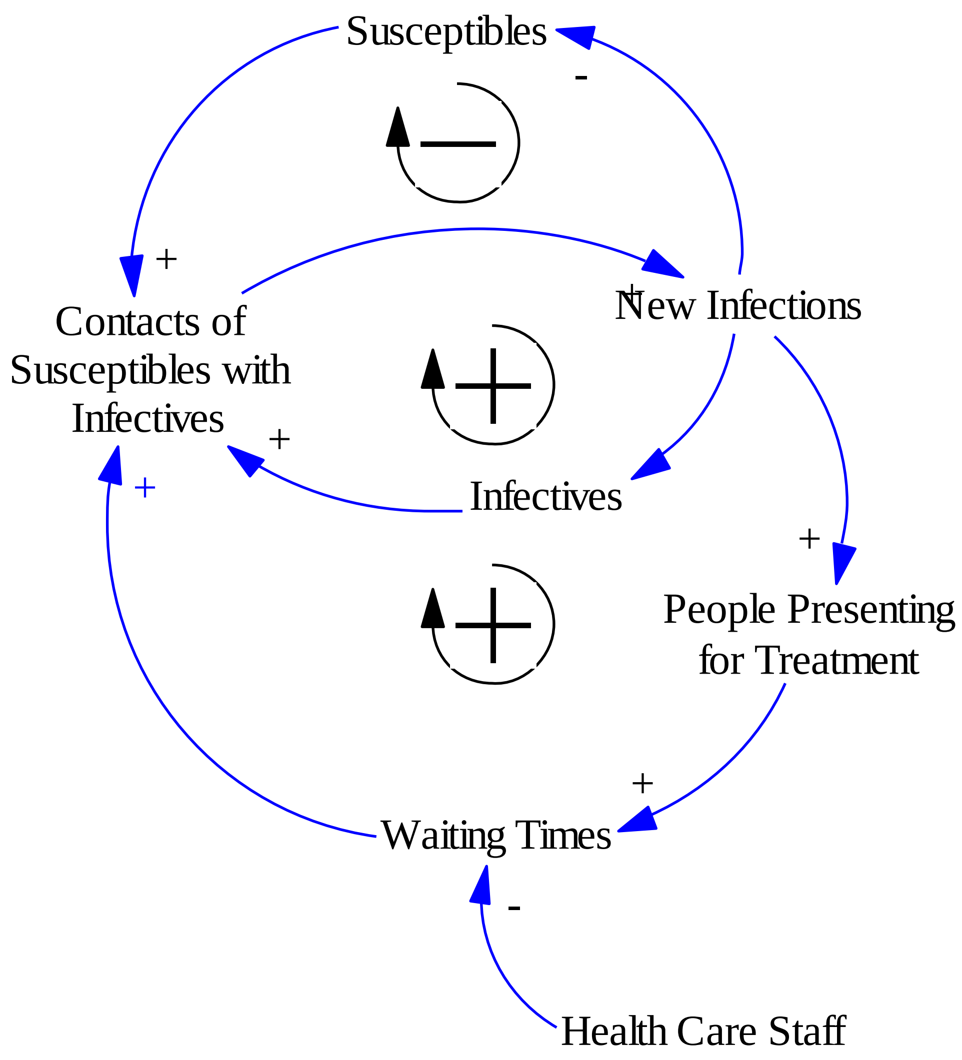


State variables over time
200,000 Person
20,000 Person
100,000 Person
150,000 Person
15,000 Person
75,000 Person
100,000 Person
10,000 Person
50,000 Person
50,000 Person
5,000 Person
25,000 Person
0 Person
0 Person
0 Person
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
Time (months)
S : Alternative 30 HC Workers Exogenous Recovery Delay Person
I : Alternative 30 HC Workers Exogenous Recovery Delay Person
R : Alternative 30 HC Workers Exogenous Recovery Delay Person

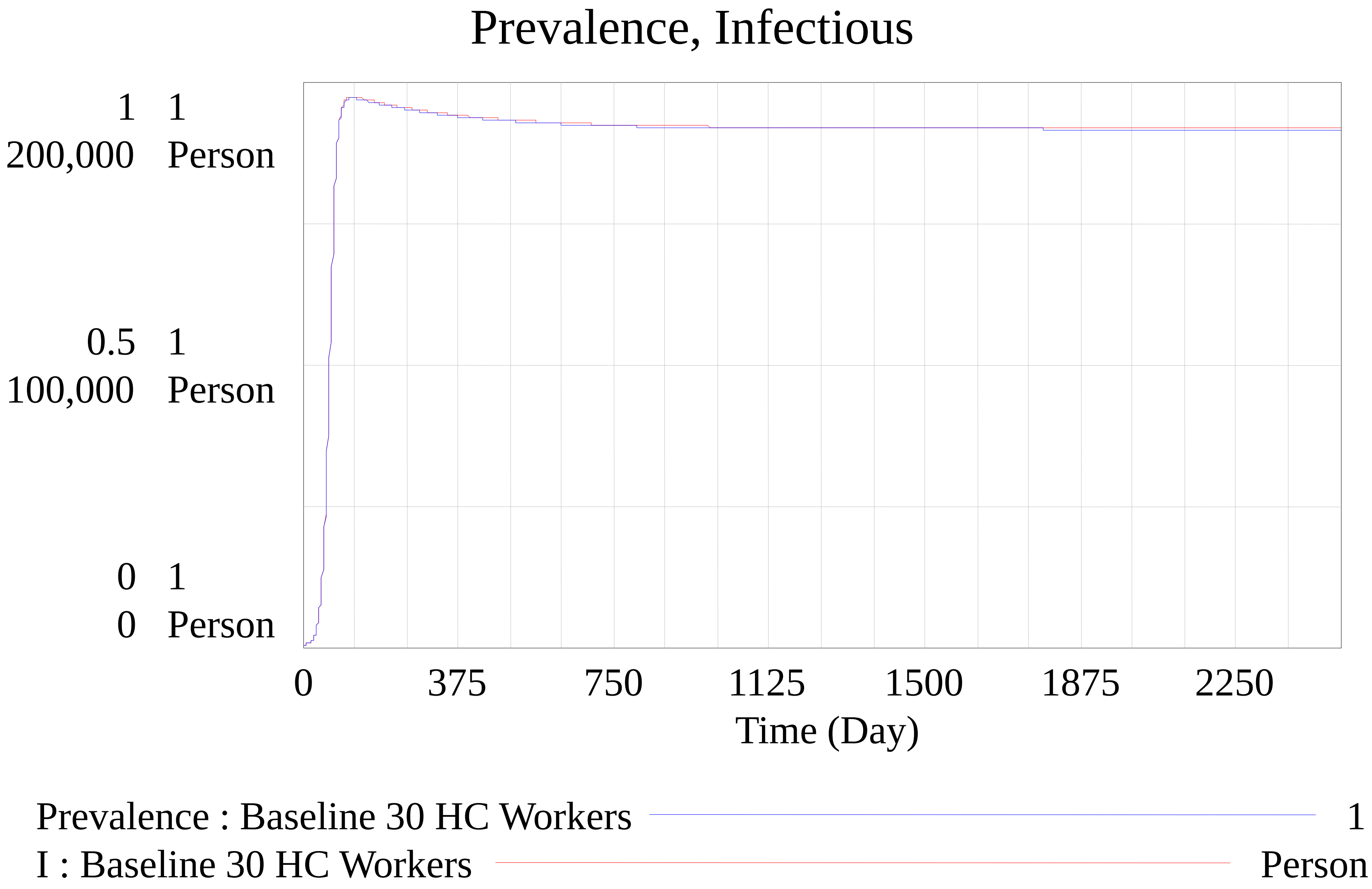
Broadening the Model Boundaries: Endogenous Recovery Delay



Broadening the Model Boundaries: Endogenous Recovery Delay



A Different Behaviour Mode



Structure as Shaping Behaviour

- System structure is defined by
  - Stocks
  - Flows
  - Connections between them
- Nonlinearity: The behaviour of the whole is more than the sum of the behaviour of the parts
  - "Emergent" behaviour would not be anticipated from simple behaviour of each piece in turn
- Stock and flow structure (including feedbacks) of a system determines the qualitative behaviour modes that the system can take on