

Sensitivity Analysis

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Using Modeling to Prepare for Changing
Healthcare Needs

Duke-NUS

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Types of Sensitivity Analyses

- Variables involved
 - One-way
 - Multi-way
- Type of component being varied
 - Parameter sensitivity analysis: Parameter values
 - Structural sensitivity analysis: Examine effects of model *structure* on results
- Type of variation
 - Single alternative values
 - Monte Carlo analyses: Draws from probability distributions (many types of variations)
- Frequency of variation
 - Static (parameter retains value all through simulation)
 - Ongoing change: Stochastic process
 - Accomplished via Monte-Carlo analyses
 - Key for DES & ABM

Model Uncertainty

- Here, we are frequently examining the impact of changing
 - Our assumptions about “how the system works”
 - Our decision of how to abstract the system behaviour
- Structural sensitivity analyses
 - Vary structure of model & see impact on
 - Results
 - Tradeoffs between choices
 - Frequently recalibrate the model in this process
- Here, we are considering uncertainty about how the current state is mapped to the next state

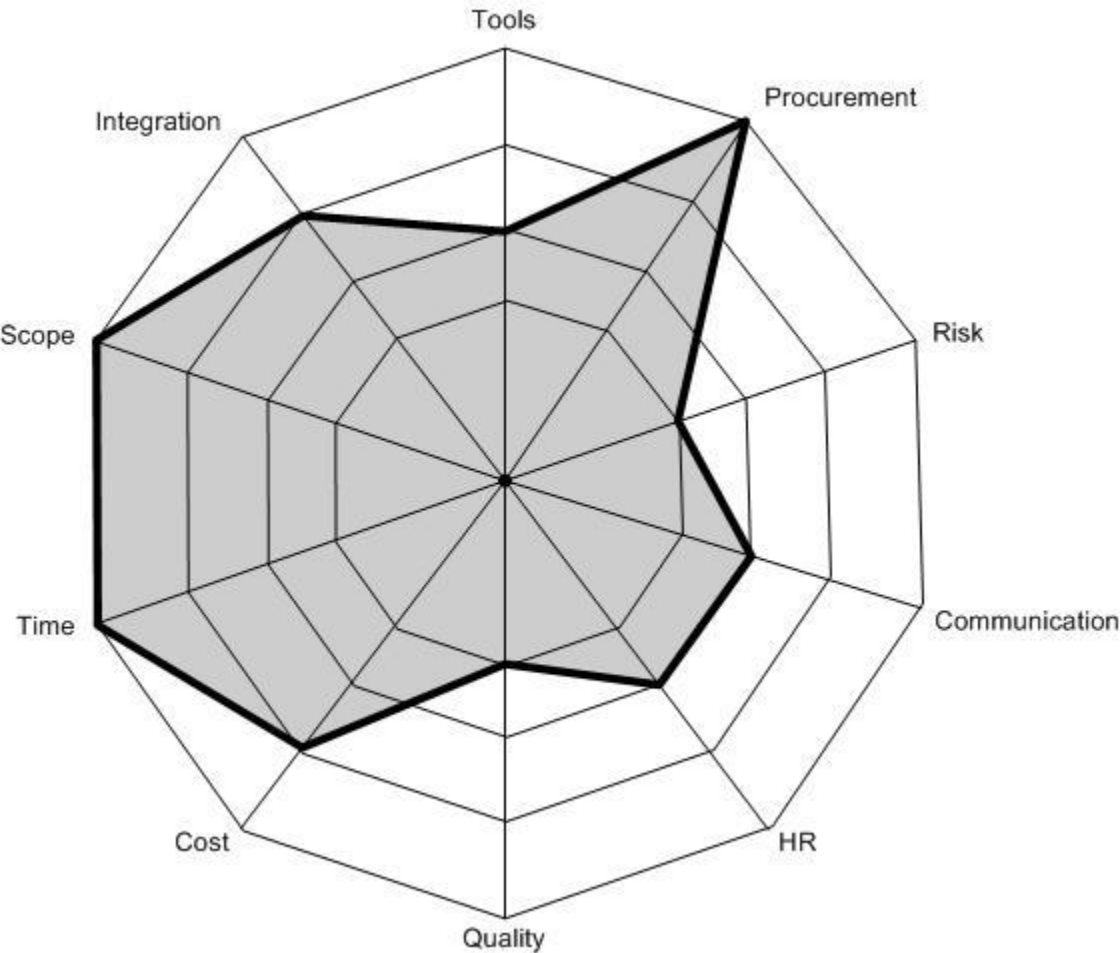
Predictor-Corrector Methods: Dealing with an Incomplete Model

- Some approaches (e.g. Kalman filter, Particle Filter) are motivated by awareness that models are incomplete
- Such approaches try to adjust model state estimates on an ongoing basis,
 - Given uncertainty about model predictions
 - New observations
- Assumption here is that the error in the model is defined by some probability distribution

Static Uncertainty Sensitivity Analyses

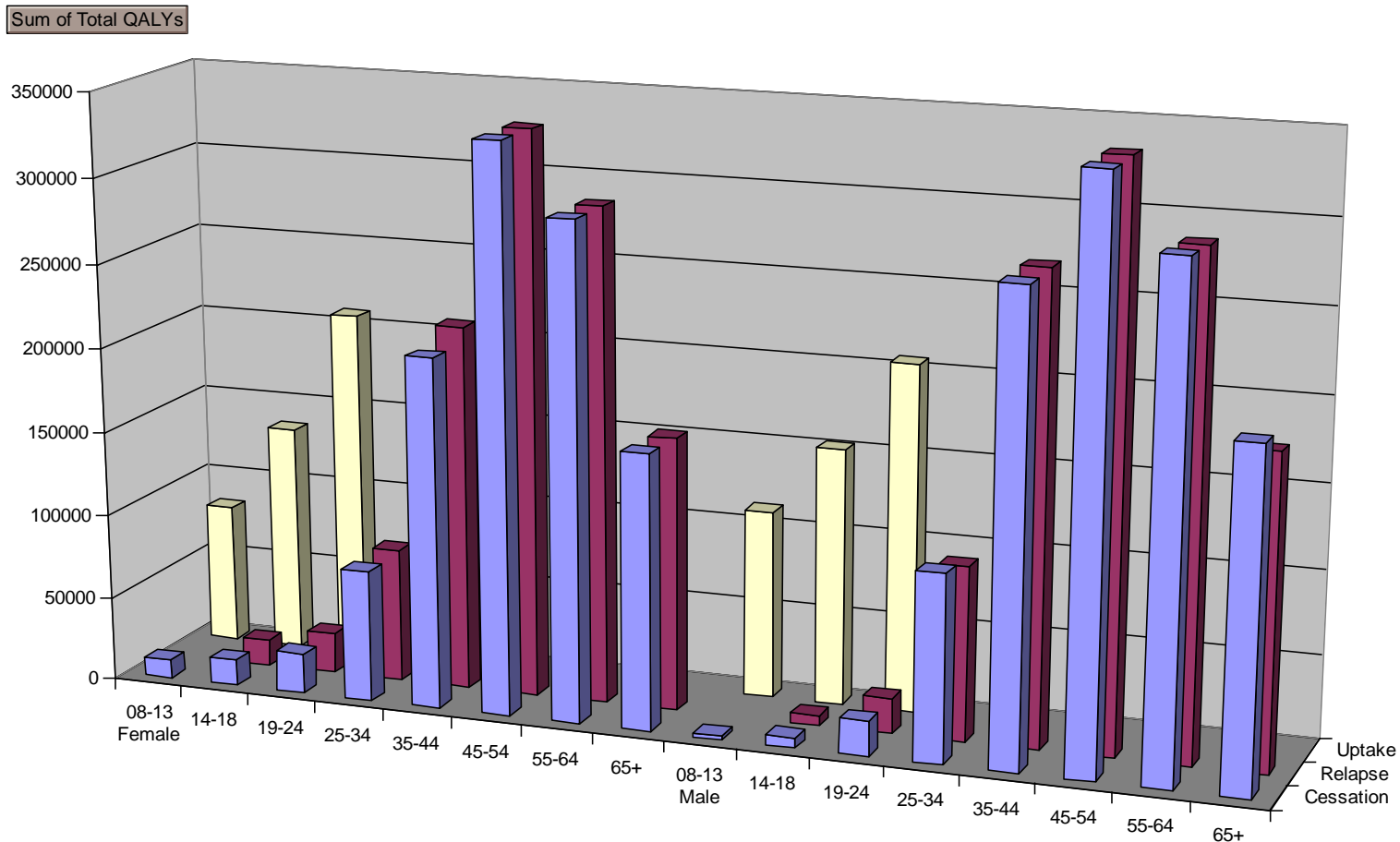
- In variation, one can seek to investigate different
 - Assumptions
 - Policies
- Same relative or absolute uncertainty in different parameters may have hugely different effect on outcomes or decisions
- Help identify parameters/initial states that strongly affect
 - Key model results
 - Choice between policies
- We place more emphasis in parameter estimation & interventions into parameters exhibiting high sensitivity

Spider Diagram



- Each axis represents a % change in a particular parameter
 - This proportional change is identical for the different parameters
- The distance assumed by the curve along that axis represents the magnitude of response to that change
 - Note that these sensitivities will depend on the state of system!

Systematic Examination of Policies



Tengs, Osgood, Lin



Hands on Model Use Ahead



Load Sample Model:

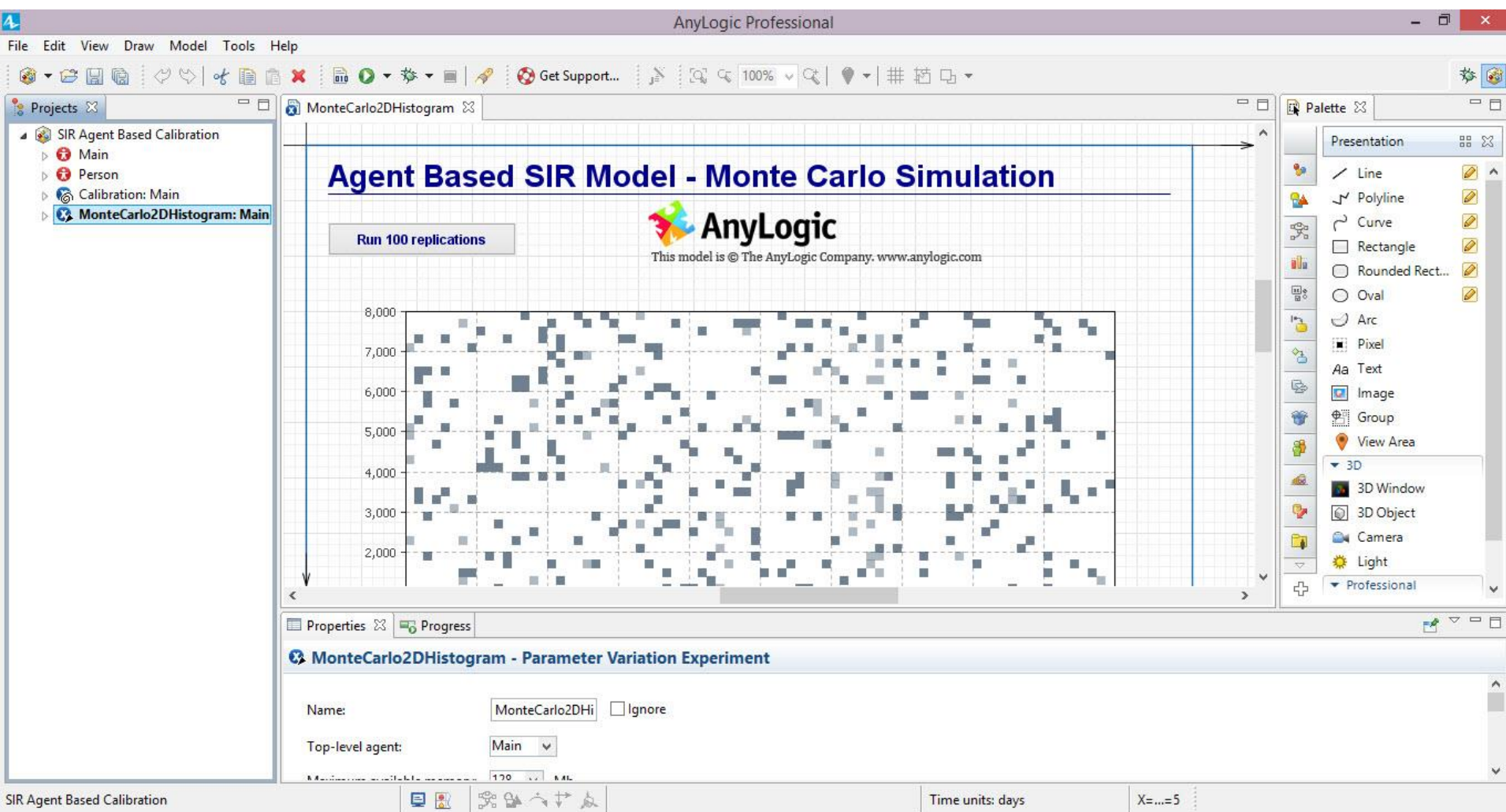
SIR Agent Based Calibration

(Via “Example Models” under “Help” Menu)

Click on “MonteCarlo2DHistogram” Experiment

Setting Ranges for Parameter Variation

Can Handle 1-Way or (Orthogonal) Multi-Way



Indicating to Systematically Vary “AverageIllnessDuration”

The screenshot displays the AnyLogic Professional interface for a Monte Carlo simulation titled "Agent Based SIR Model - Monte Carlo Simulation". The main workspace shows a histogram of simulation results with a title "Run 100 replications" and the AnyLogic logo. Below the workspace, the "Properties" panel is open, showing the "MonteCarlo2DHistogram - Parameter Variation Experiment" settings. A red arrow points to the "Parameters" section, specifically to the "Varied in range" radio button, which is highlighted in a red box. A red text box on the left side of the image contains the instruction "Click on 'Varied in range'", with a red arrow pointing to the "Varied in range" radio button.

Parameters

Parameters: ☒ Varied in range ☐ Freeform

Number of runs: 200

Parameter	Type	Value		
		Min	Max	Step
Average...ration	Fixed	15		
ContactRate	Fixed	1.0		
Infectio...ability	Fixed	0.8		
TotalPopulation	Fixed	10000		

Time units: days X=...97

Entering Extremes & Step of the Variation

AnyLogic Professional

File Edit View Draw Model Tools Help

Projects

- SIR Agent Based Calibration*
- Main
- Person
- Calibration: Main
- MonteCarlo2DHistogram: Main

MonteCarlo2DHistogram

Agent Based SIR Model - Monte Carlo Simulation

Run 100 replications

AnyLogic

This model is © The AnyLogic Company. www.anylogic.com

8,000

7,000

Properties Progress

MonteCarlo2DHistogram - Parameter Variation Experiment

Parameters: ☒ Varied in range ☐ Freeform

Number of runs: 200

Parameter	Type	Value
		Min Max Step
Average...ration	Range	5 25 5
ContactRate	Fixed	1.0
Infectio...ability	Fixed	0.8
TotalPopulation	Fixed	10000

Model time

Use calendar

Stop: top at specified time

Time units: days

X=...93

5 possible values for InfectionProbability { 5, 10, 15, 20, 25 }

Note: Simulation Count is Defined Implicitly

The screenshot displays the AnyLogic Professional interface. The main workspace shows a simulation titled "Agent Based SIR Model - Monte Carlo Simulation" with a "Run 100 replications" button. Below the workspace, the "Properties" panel is open, showing the "MonteCarlo2DHistogram - Parameter Variation Experiment" configuration. In this panel, the "Parameters" section has "Varied in range" selected, and the "Number of runs" is set to 200. A red arrow points from a text note to this "Number of runs" field. Below the parameters, a table lists the varied parameters: Average...ration, ContactRate, Infectio...ability, and TotalPopulation. The "Model time" section at the bottom shows "Use calendar" unchecked and "Stop:" set to "Stop at specified time".

MonteCarlo2DHistogram - Parameter Variation Experiment

Parameters: ☒ Varied in range ☐ Freeform

Number of runs: 200

Parameter	Type	Value	Min	Max	Step
Average...ration	Range		5	25	5
ContactRate	Fixed	1.0			
Infectio...ability	Fixed	0.8			
TotalPopulation	Fixed	10000			

Model time

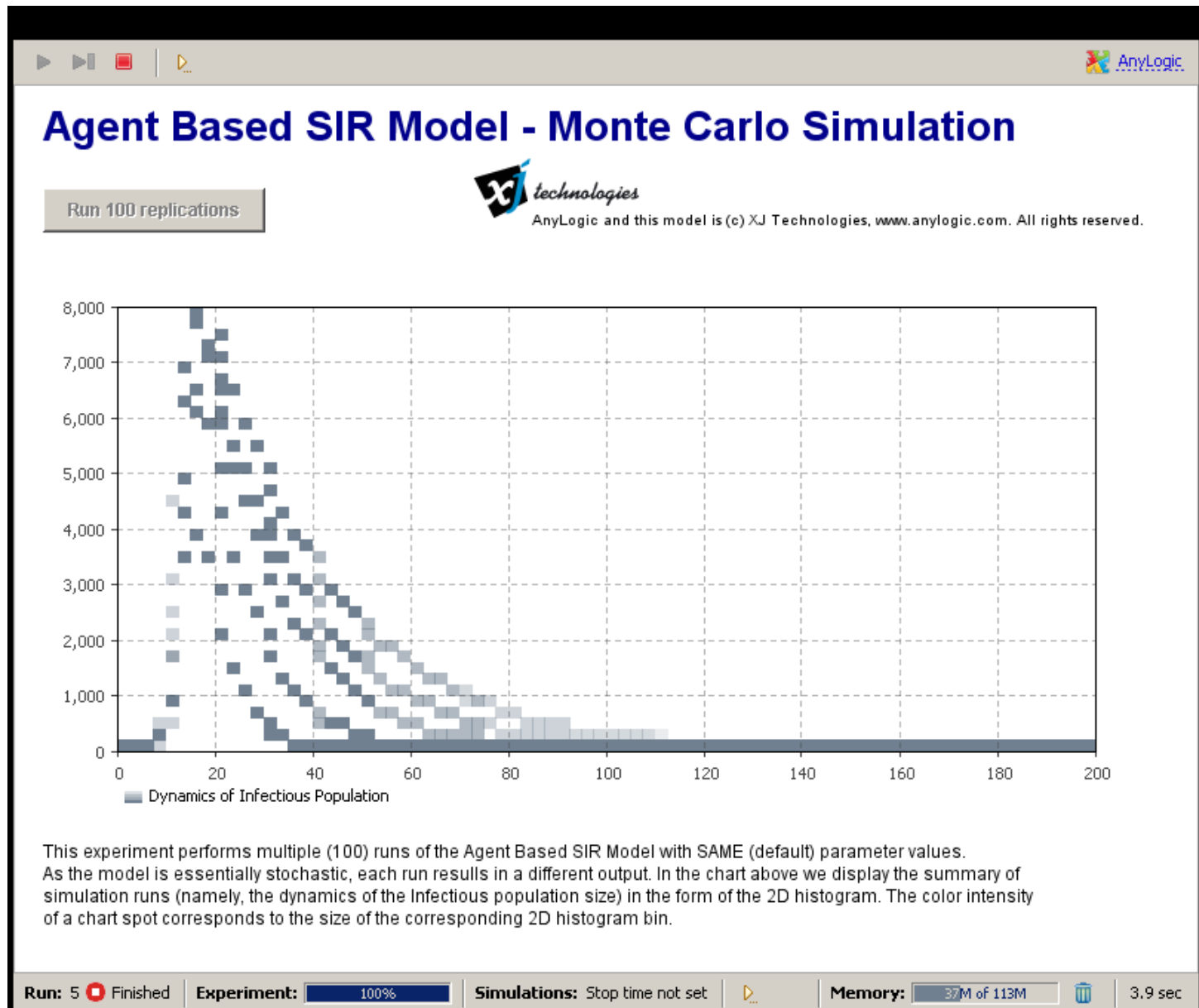
☐ Use calendar

Stop: Stop at specified time

Time units: days X=...93

Note that one loses the capacity To explicitly set Simulation count when using param. ranges

“1 Way” Sensitivity Exploration in AnyLogic



Multi-Way Sensitivity Analyses

- When examining the results of changing multiple variables, need to consider how multiple variables vary together
- If this covariation reflects dependence on some underlying factor, may be able to simulate uncertainty in underlying factor

Set Range for InfectionProbability

11 possible values for InfectionProbability
{
0,0.1,0.2,.3,.4,.5,.6,7,8,9,10
}

AnyLogic Professional

File Edit View Draw Model Tools Help

Projects

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MonteCarlo2DHistogram

Agent Based SIR Model - Monte Carlo Simulation

Run 100 replications

AnyLogic

This model is © The AnyLogic Company. www.anylogic.com

Properties Progress

MonteCarlo2DHistogram - Parameter Variation Experiment

Parameters: ☒ Varied in range ☐ Freeform

Number of runs: 200

Parameter	Type	Value
		Min Max Step
Average...ration	Range	5 25 5
ContactRate	Fixed	1.0
Infectio...ability	Range	0 1 0.1
TotalPopulation	Fixed	10000

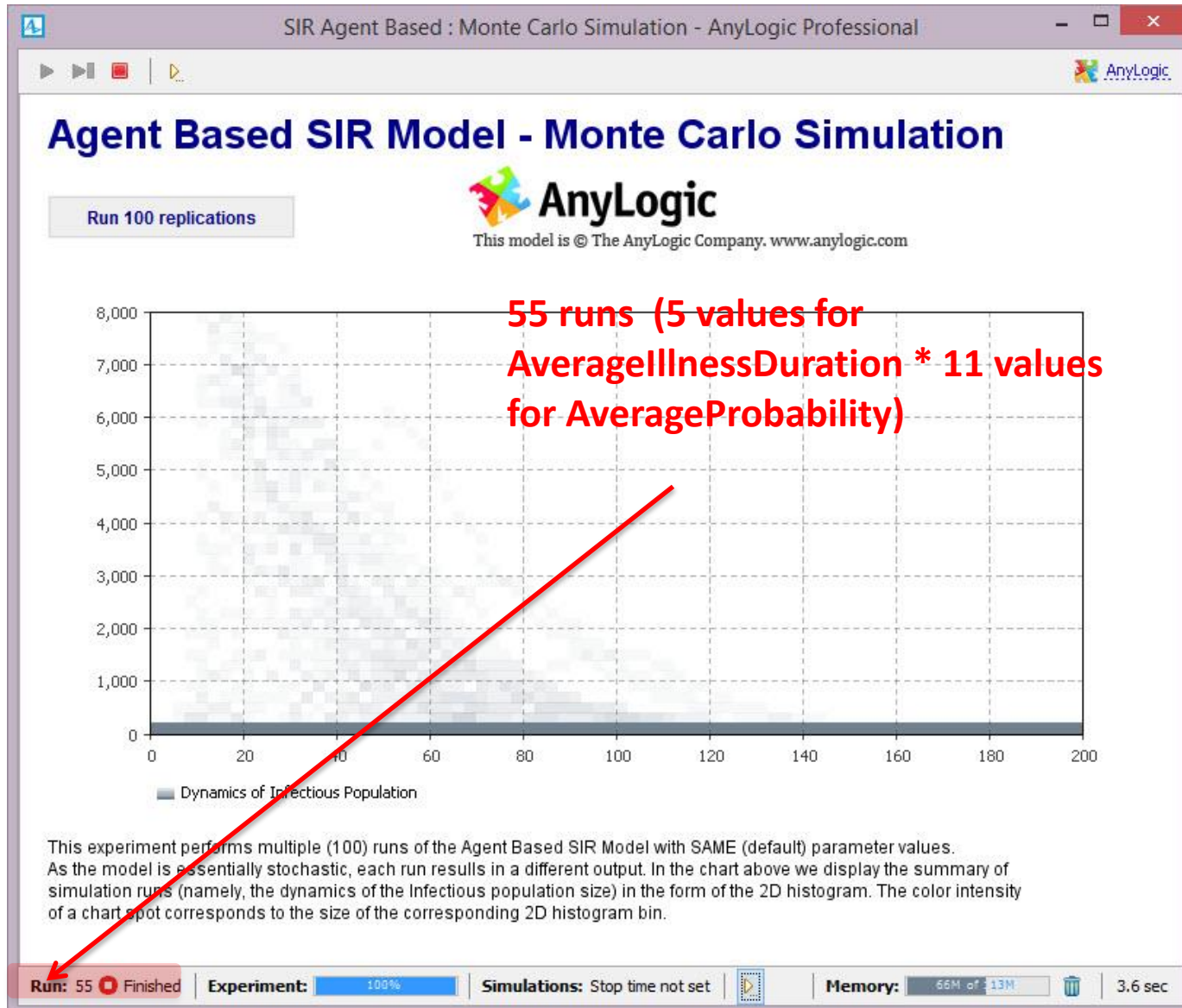
Model time

☐ Use calendar

Stop: top at specified time

Time units: days X=...78

Resulting Output



Challenge: Combinatorial Explosion

- If we have n distinct parameters, each varying over c values, we have c^n possible combinations of parameter values
- As c and (far more important) n rise, it quickly becomes infeasible to exhaustively examine such combinations
- Alternatives
 - Dimensional analysis
 - Techniques for judiciously selecting combinations count
 - Monte Carlo techniques

Challenge: Combinatorial Explosion

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Dimensional Analysis

- Observation: Often it is not a parameter in isolation that matters, but combinations of parameters
 - Ratio of two parameters (speed of infection spread vs. speed of intervention)
 - Multiplication of two parameters (βc)
- Dimensional analysis techniques exploit the reflection that our choice of units has no impact on how the world operates to recognize that a process must be governed by **dimensionless quantities**
- We can typically reduce the parameter count by
 - Determining dimensions associated w/model quantities
 - Determining a set of dimensionless quantities

Challenge: Combinatorial Explosion

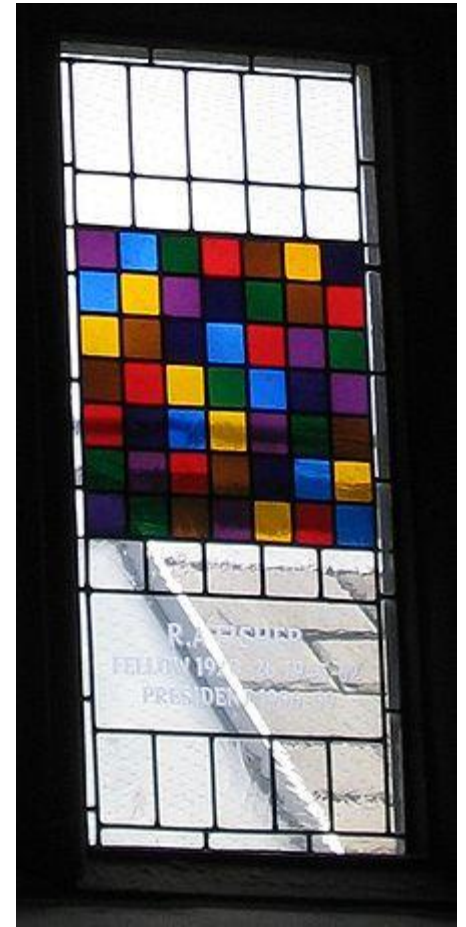
- If we have n distinct parameters, each varying over c values, we have c^n possible combinations of parameter values
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Judiciously Selecting Set of Parameter Combinations Being Examined: Discrete

- Latin hypercubes
 - Test each value of each parameter at least once
 - i.e. for each possible value of a given parameter p , we are guaranteed to explore at least one parameter vector where parameter p holds that value
- Orthogonal arrays
 - Test each pair of parameter values at least once
 - i.e. for each possible pair of values of given parameters p_1 and p_2 , we are guaranteed to explore at least one parameter vector where parameters p_1 and p_2 hold those values

Latin Squares

- Each value (or equivalence class) of each field occurs exactly once
- This is a “minimal set” that exercises all values (equivalence classes) of each field



Orthogonal Arrays (“Pair-wise Testing”, “All Pairs Testing”)

- Key characteristic: Every *pair* of variable values occurs at least once (but not all combinations!)
 - NB:contrast with Latin Squares, where every value is tried
- Exploits fact that *compatibility* problems typically are pairwise
 - If problem is not revealed in just by single value or equivalence class, it is likely to be no more than a *pair*
- The count of all combinations of values would typically be vastly larger
 - All Combinations: $\prod_{i=1}^n |X_i|$ Rises as least as 2^n
 - Pairwise: $\max(|X_i|) \max_{j,j \neq i} (|X_j|)$ (?)
- => we can do much less work than required for all combinations, but still secure most of value!

Example Case Reduction

S.No	Parameters and Values	100% coverage	reduced to
1	$3^2 2^2 1^1$	36	9
2	$10^4 7^4 8^3 6^2 3^2 5^2 4^2 9^1$	14338695168000000	144
3	10^{15}	10000000000000000	199
4	$10^5 5^6 3^5 2^5$	24300000000000	142
5	$3^3 4^2 5^2 6^2 7^1 8^2 10^1 9^1$	15676416000	96

Judiciously Selecting Set of Parameter Combinations Being Examined: Continuous

- Grid
- Grid adaptation of
 - Latin Hypercube
 - Orthogonal array
- While not directly sampled by AnyLogic, these can be easily implemented by
 - Creating vectors of the appropriate values to use successively for a given parameter value
 - For a given iteration, finding the appropriate value for a particular parameter by indexing into the vector for that parameter

Challenge: Combinatorial Explosion

- If we have n distinct parameters, each varying over c values, we have c^n possible combinations of parameter values
- As c and (far more important) n rise, it quickly becomes infeasible to exhaustively examine such combinations
- Alternatives
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Imposing a Probability Distribution

Monte Carlo Analysis

- We feed in probability distributions to reflect our uncertainty about one or more parameters
- The model is run many, many times (realizations)
 - For each realization, the model uses a different draw from those probability distribution
- What emerges is resulting probability distribution for model outputs

Monte Carlo Techniques

(Continuous Parameters)

- For a given amount of computational effort, Monte Carlo techniques typically exhibit far more favourable properties as the count of dimensions rises than does downsampling the grid of values
- Effort Scaling
 - Grid: $\theta\left(\prod_{i=1}^D d_i\right)$
 - Monte Carlo: $\theta(N)$ (N: count of Monte Carlo samples)
- Error (eg when determining mean) given limited eff.
 - Coarser grid: $\theta\left(c^{-\frac{2}{D}}\right) = \theta\left(\frac{1}{c^{\frac{2}{D}}}\right)$ Spends much effort in low-probability areas of space
 - Monte Carlo: $\theta\left(\frac{1}{\sqrt{N}}\right)$ Naturally concentrates samples in highly probable regions (because sampled)

Monte Carlo Analyses in AnyLogic:

Specifying Distributions for Parameters

The screenshot displays the AnyLogic Professional software interface. The main workspace shows a diagram titled "Agent Based SIR Model - Monte Carlo Simulation" with a button labeled "Run 100 replications". The interface includes a menu bar (File, Edit, View, Draw, Model, Tools, Help), a toolbar, and a palette on the right with various drawing tools.

The "Properties" panel is open, showing the "MonteCarlo2DHistogram - Parameter Variation Experiment" configuration. The "Parameters" section is expanded, showing the following settings:

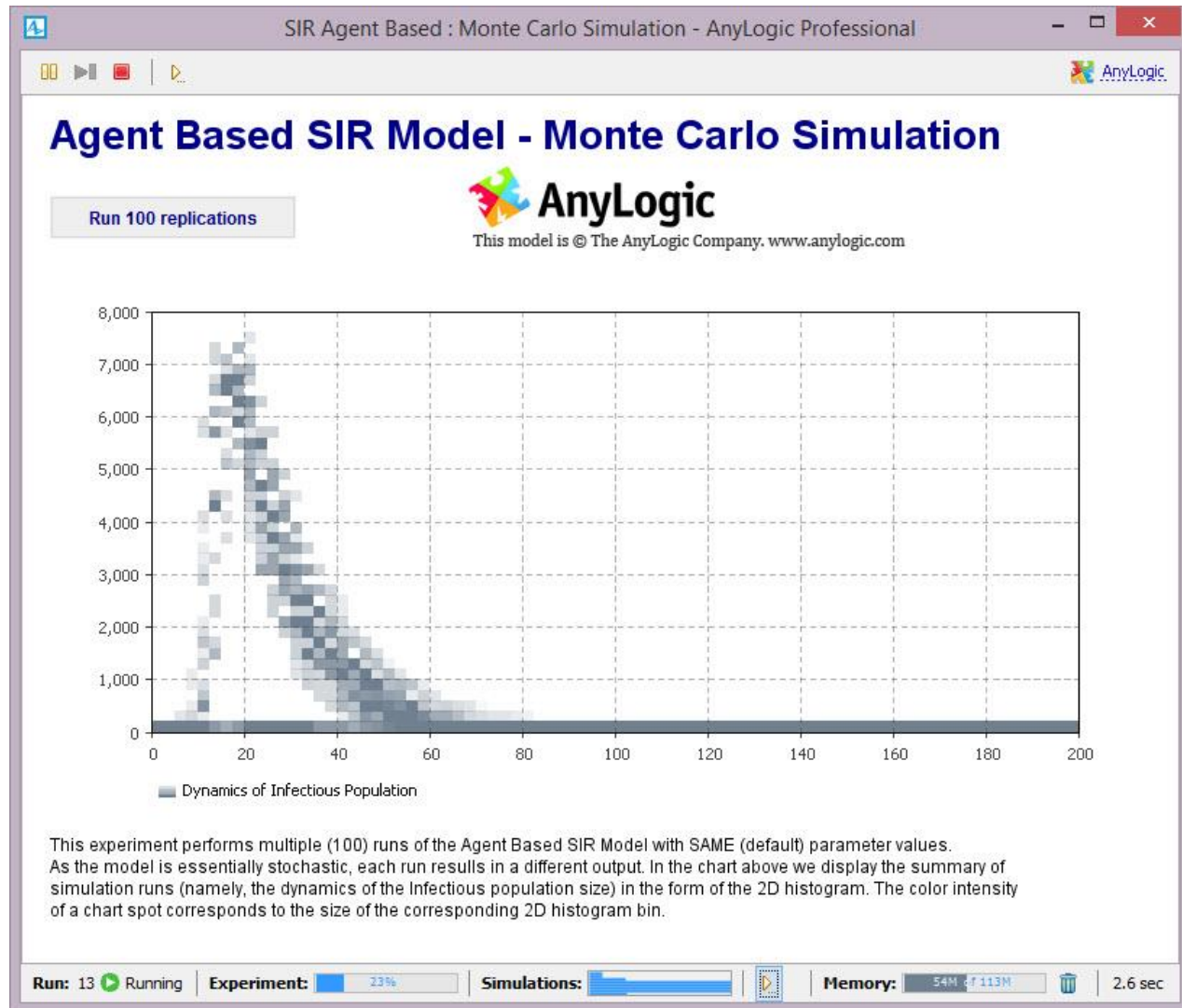
- Name: MonteCarlo2DHi ☐ Ignore
- Top-level agent: Main
- Maximum available memory: 128 Mb
- Create default UI
- Parameters: ☐ Varied in range ☒ Freeform
- Number of runs: 100

The "Parameters" table is also visible:

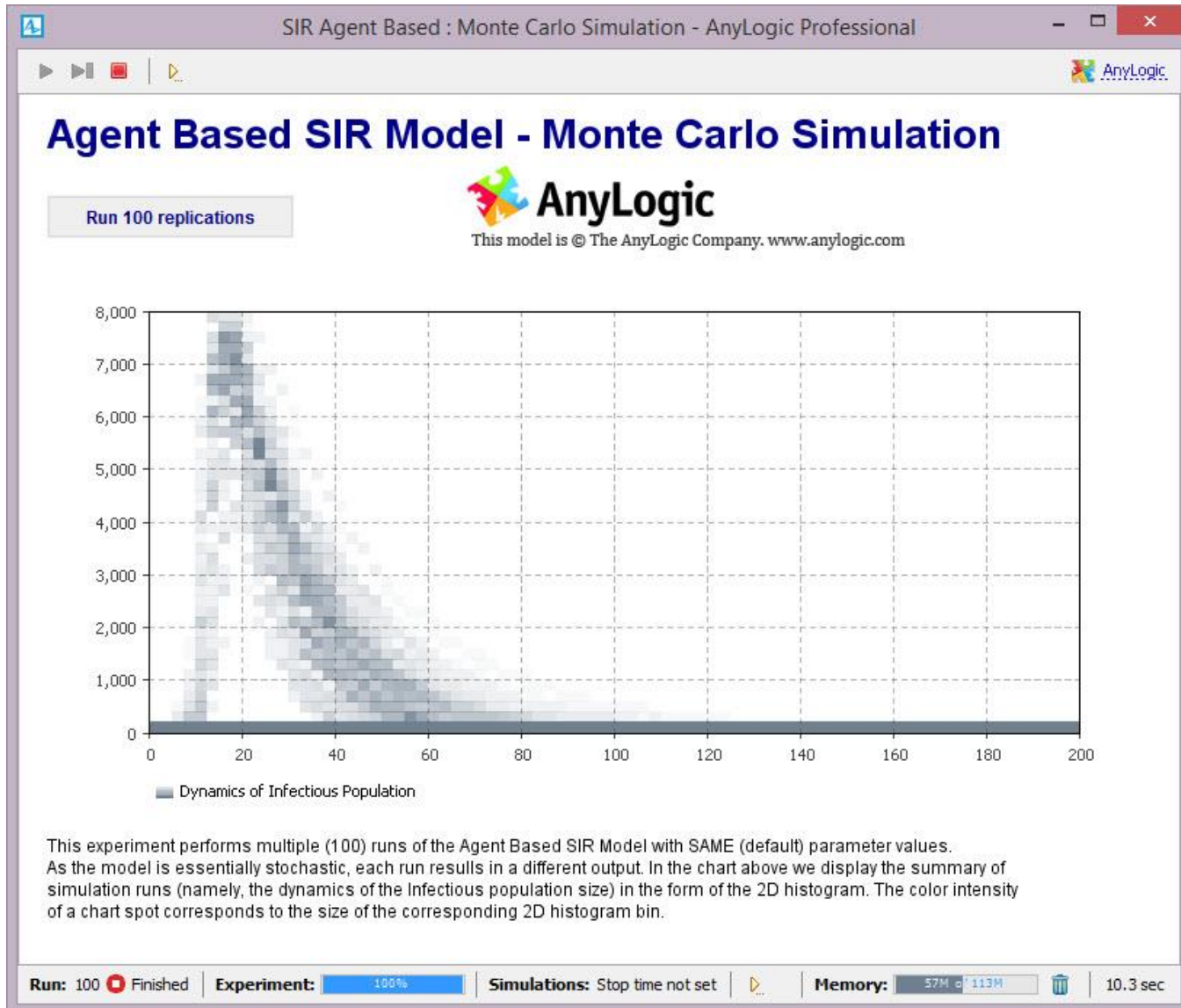
Parameter	Expression
Average...ration*	max(0,normal(5,15))
ContactRate	1.0
Infectio...ability	0.8
TotalPopulation	10000

The status bar at the bottom indicates "Time units: days" and "X=...01".

Monte Carlo Output After Some Runs



Monte Carlo Output After All Runs



Populating the 2D Dataset

The screenshot displays the AnyLogic Professional software interface. The main workspace shows a diagram titled "Agent Based SIR Model - Monte Carlo Simulation" with a button labeled "Run 100 replications". The diagram includes the AnyLogic logo and the text "This model is © The AnyLogic Company. www.anylogic.com".

The left sidebar shows the project structure:

- SIR Agent Based Calibration*
 - Main
 - Person
 - Calibration: Main
 - MonteCarlo2DHistogram: Main

The right sidebar shows the Palette with various shapes and lines:

- Presentation
 - Line
 - Polyline
 - Curve
 - Rectangle
 - Rounded Rect...
 - Oval

The bottom section shows the "MonteCarlo2DHistogram - Parameter Variation Experiment" configuration:

- ☒ Memory
- Java actions
 - Initial experiment setup:
 -
 - Before each experiment run:
 - `dataInfectious2D.reset();`
 - Before simulation run:
 -
 - After simulation run:
 - `dataInfectious2D.add(root.InfectiousDS);`
 - After iteration:
 -
 - After experiment:
 -

The status bar at the bottom indicates "Time units: days" and "X=...27".

Sensitivity in Initial States

- Frequently we don't know the exact state of the system at a certain point in time
- A very useful type of sensitivity analysis is to vary the initial model state
- In aggregate models, this can be accomplished by
 - Varying the number of people in the stock via a parameter to adjust
- In an agent-based model, state has far larger dimensionality
 - Can modify different numbers of people with characteristic, location of people with characteristic, etc.