A GENERAL APPROACH TO MUSCLE WRAPPING OVER MULTIPLE SURFACES

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INTRODUCTION

In musculoskeletal simulations muscle forces are transmitted along curves intended to represent the centroid of each muscle's path. The curves are assumed to take the shortest path wrapping around geometric surfaces that represent bones and other structures. The length and rate of length change of a muscle's path affect computed muscle force and the geometry of a muscle's path determines how muscle force is delivered to bones. Errors in representing how muscles wrap may degrade simulation accuracy and performance.

A common approach to represent muscle wrapping uses approximate discretized wrapping curves [1]. In dynamics simulations, discontinuous changes in wrapping paths due to discretization can degrade simulation performance. Another approach to model muscle wrapping uses analytical equations for simple shapes such as spheres and cylinders [2, 3]. This approach does not generalize to muscle paths that wrap around more than two surfaces or complex wrapping surfaces.

Here we introduce a novel formulation to compute smooth wrapping curves for arbitrary numbers of wrap surfaces. The formulation permits the use of general smooth geometric surfaces with implicit or parametric representations and incorporates fast analytical equations for the special cases of simple shapes. This method generates smooth wrapping paths suitable for high-order time integration, and allows biomechanical models of the spine, finger, shoulder, and other systems to incorporate wrapping paths over multiple anatomical structures with complex shapes.

METHODS

Our formulation computes the minimum length wrapping path over multiple surfaces by finding the location of two wrapping points per surface such that joining paths are straight lines, wrapping paths are geodesic curves, and joining paths connect smoothly with wrapping paths (Figure 1).

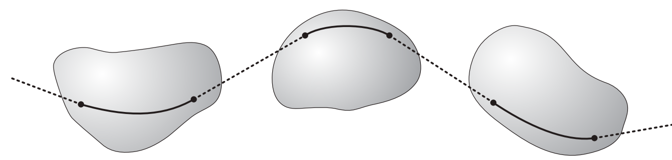

Figure 1: The shortest path wrapping n surfaces is represented with two wrapping points on each surface, straight joining paths (dotted lines), and geodesic wrapping paths (solid lines).

A geodesic curve is nominally the shortest path along a smooth surface and is uniquely defined for a point and direction on that surface. Our formulation ensures that the path remains smooth as wrapping paths lift off wrapping surfaces during dynamic simulations. In the case of an implicit surface representation, $\phi(\boldsymbol{r}) = 0$, with gradient, $\boldsymbol{n} \equiv \nabla\phi$, we aim to find the location of wrapping points, $\boldsymbol{p}$ and $\boldsymbol{q}$, that satisfy the conditions illustrated in Figure 2.

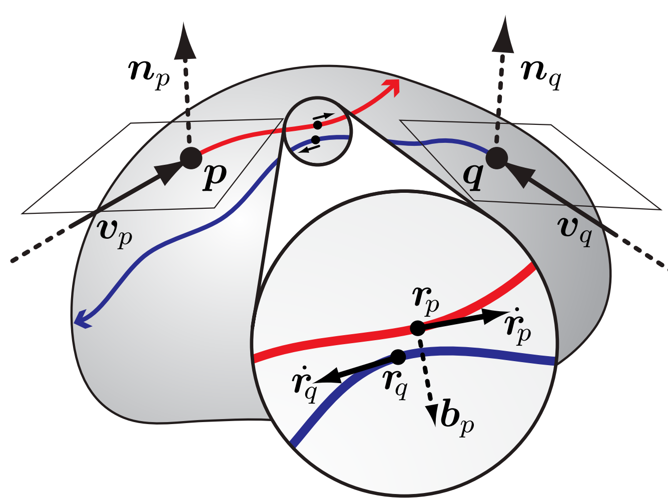

Figure 2: The local conditions that must be satisfied to find the shortest wrapping path. Wrapping points $\boldsymbol{p}$ and $\boldsymbol{q}$ must lie on the wrapping surface and straight-line joining paths $\boldsymbol{v}_p$ and $\boldsymbol{v}_q$ must be tangent to the surface. The geodesic curve originating from point $\boldsymbol{p}$ in the direction of $\boldsymbol{v}_p$ (red line) and the geodesic curve originating from point $\boldsymbol{q}$ in the direction $\boldsymbol{v}_q$ (blue line) must connect smoothly.

The wrapping conditions can be stated as follows:

1. Wrapping points lie on the wrapping surface:

$$\begin{aligned} \phi(\boldsymbol{p}) &= 0 \\ \phi(\boldsymbol{q}) &= 0 \end{aligned} \tag{1}$$

2. Joining paths are tangent to the wrapping surface:

$$\begin{aligned} \boldsymbol{n}_p \cdot \boldsymbol{v}_p &= 0 \\ \boldsymbol{n}_q \cdot \boldsymbol{v}_q &= 0 \end{aligned} \tag{2}$$

where $\boldsymbol{n}$ is the surface normal ($\boldsymbol{n} \equiv \nabla\phi$ for implicit surface representations).

3. The geodesic curves originating from the two wrapping points must connect smoothly:

$$\begin{aligned} \boldsymbol{b}_p \cdot (\boldsymbol{r}_q - \boldsymbol{r}_p) &= 0 \\ \boldsymbol{b}_p \cdot \dot{\boldsymbol{r}}_q &= 0 \end{aligned} \tag{3}$$

where $\boldsymbol{b}_p \equiv \dot{\boldsymbol{r}}_p \times \boldsymbol{n}_p$, and $\boldsymbol{r}_p$ and $\boldsymbol{r}_q$ are the closest points on the geodesic curves from $\boldsymbol{p}$ and $\boldsymbol{q}$.

We compose a system of equations using conditions (1), (2), and (3) for all n wrapping surfaces as:

$$\boldsymbol{F}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \equiv \left(\boldsymbol{p}_1^T \; \boldsymbol{q}_1^T \; \ldots \; \boldsymbol{p}_n^T \; \boldsymbol{q}_n^T\right)^T \tag{4}$$

The size of the system is $6\,n_{\text{implicit}} + 4\,n_{\text{parametric}}$, since condition (1) is automatically satisfied for parametric surface representations. We solve (4) as a root-finding problem using Newton's method with a banded Jacobian to find wrapping point locations across all n surfaces. Fast convergence requires good initial conditions; to achieve this we take advantage of temporal coherence for wrapping in dynamic simulations.

For general smooth surfaces, we find geodesic curves using numerical integration. For this purpose, we use the mechanical analogy that a particle moving along a surface with acceleration normal to the surface will trace a geodesic path [4]. For a particle $\boldsymbol{r}$, we solve the differential equation $\boldsymbol{M}\ddot{\boldsymbol{r}} = -\boldsymbol{G}^T\lambda$ subject to $\phi(\boldsymbol{r}) = 0$, which leads to:

$$\begin{pmatrix} \boldsymbol{M} & \boldsymbol{G}^T \\ \boldsymbol{G} & 0 \end{pmatrix} \begin{pmatrix} \ddot{\boldsymbol{r}} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ -\dot{\boldsymbol{G}}\dot{\boldsymbol{r}} \end{pmatrix}$$

where $\boldsymbol{G}^T \equiv \boldsymbol{n}$ and λ is the Lagrange multiplier that satisfies the implicit surface constraint.

For spheres, cylinders and other surfaces of revolution, it is straightforward to replace the general geodesic condition (3) with analytical equations to improve speed.

RESULTS AND DISCUSSION

We evaluated the accuracy of our approach using a combined sphere-cylinder test case reported in [3]. We chose initial wrapping points on the straight-line path between the fixed end points (Figure 3, dotted line). Our computed wrapping path (Figure 3, solid line) has a length of 22.437 cm, which matches the exact solution reported in [3]. We also simulated test cases for wrapping multiple bicubic spline surfaces, ellipsoids, spheres, and cylinders as shown in Figure 4. Videos and source code are available at http://simtk.org/home/wrap. This method provides high performance muscle wrapping for simulation of musculoskeletal dynamics.

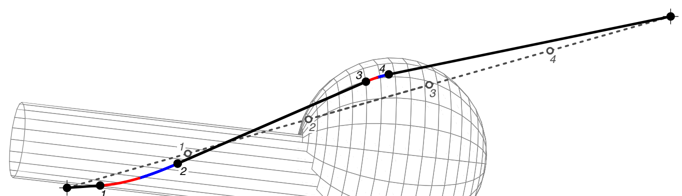

Figure 3: Result for wrapping over the sphere-cylinder test case from [3]. The initial path is denoted by the dotted line.

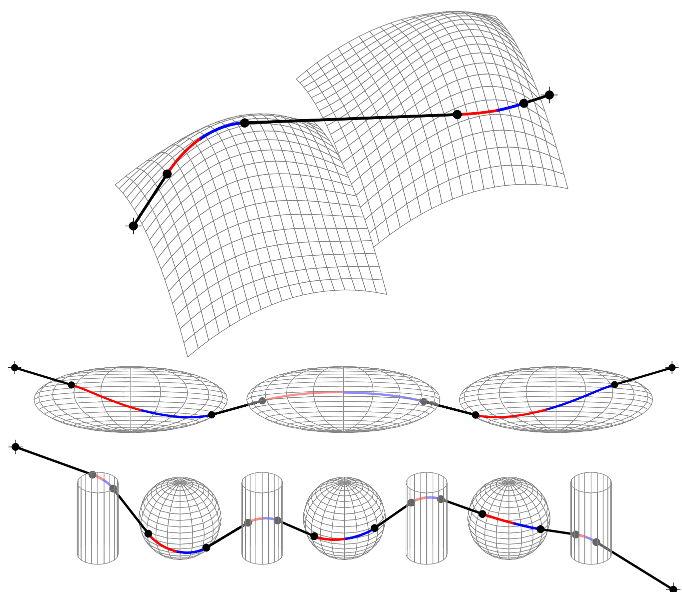

Figure 4: Results for wrapping over multiple surfaces: two bicubic spline surface patches (top), three ellipsoids (middle), and seven spheres and cylinders (bottom).

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