Parallel Numerical Solutions of the QCD Bethe-Salpeter Equation II

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The Bethe-Salpeter Equation

- The purpose of the project is to model 2-body bound states in quantum chromodynamics (QCD).
- The wavefunction of a bound state can be obtained by solving the QCD Bethe-Salpeter equation:

$$\Gamma(p;P) = \int \frac{d^4q}{(2\pi)^4} K(q,p;P)\, S\left(q^+\right) \Gamma(q;P)\, S\left(q^-\right).$$

- This is an integral equation for the Bethe-Salpeter amplitude $\Gamma(p;P)$, which is a 4×4 matrix.
- The equation can be made more numerically accessible by expanding the solution in various degrees of freedom.

Expansions

- The first step is to expand the solution in Lorentz components (reducing the problem from an IE for a 4×4 solution to 4 coupled IEs for 1×1 solutions):

$$\Gamma(p;P) = \sum_{i=1}^{N} F_i(p;P)\,\widehat{\tau}_i(p;P)\,.$$

- Then a Chebyshev decomposition can be performed to reduce 4D integration to a set of coupled 1D integrations:

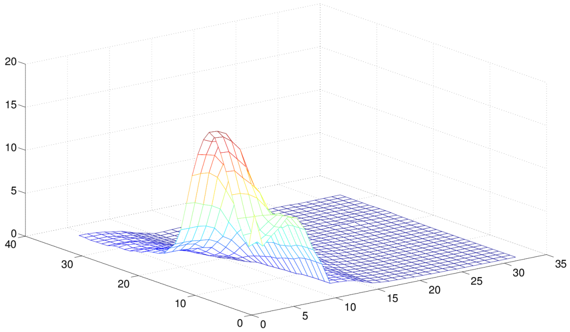
$$F_i(p,P) = \sum_{j=0}^{M} U_j(u)\,\tilde{F}_{ij}\left(p^2,P^2\right).$$

- The remaining integration can be discretized, and an artifical eigenvalue can be inserted to allow the external momentum scale P to be fixed:

$$\lambda\left(P^2\right)\tilde{F}_{ij}\left(p^2\right) = \mathscr{K}\left(q^2,p^2,P^2\right)\cdot\tilde{F}_{ij}\left(q^2\right).$$

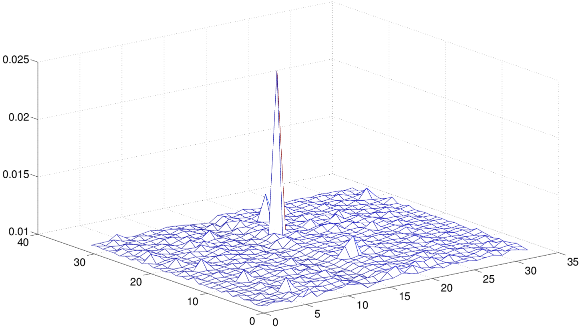
Evaluation of $\mathscr{K}\left(P^2\right)$

- Evaluating the elements of $\mathscr{K}\left(P^2\right)$ requires double integrations over the angular variables u. These integrations were attempted using both an adaptive Simpson quadrature rule and a Gauss-Legendre nonadaptive quadrature rule.
- Timings for adaptive Simpson quadrature (in seconds):



Evaluation of $\mathscr{K}(P^2)$

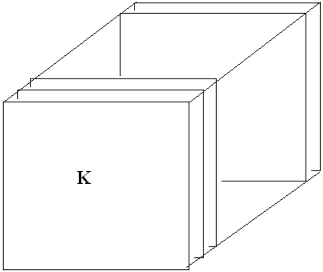
- Timings for Gauss-Legendre quadrature (in seconds):



- The two methods produce results that are the same to within 1%: $\max\left[\frac{\mathscr{K}_{Simp}-\mathscr{K}_{GL}}{\mathscr{K}_{Simp}}\right] = 0.0033$.

Parallelization of Problem

- The problem can be visualized as evaluation of a 3D array:



- Several methods were implemented on Socrates for the basic problem (32 grid size, both expansions truncated after one entry: $N = M = 1$). The following tables show minimum timings over 5 runs for 80 different values of P.

Parallelization of Problem

- These two tables are for versions which parallelize the loop over P (Table A), and the loop over the elements of $\mathscr{K}$ (Table B).

A)

p	Time (seconds)	Speedup
1	161.522337	1
2	85.508760	1.8890
4	49.311946	3.2755
8	36.629304	4.4096

B)

p	Time (seconds)	Speedup
1	166.076348	1
2	89.910633	1.8471
4	53.421731	3.1088
8	41.934506	3.9604

Parallelization of Problem

- The left table (C) makes both the loops over P and over the elements of $\mathscr{K}$ `parfor` loops.
- The right hand table (D) involves a single parallel loop over the elements of the full 3D array.

C)

p	Time (seconds)	Speedup
1	162.320255	1
2	85.886755	1.8899
4	49.494977	3.2795
8	36.357750	4.4645

D)

p	Time (seconds)	Speedup
1	163.824436	1
2	87.518920	1.8719
4	49.092138	3.3371
8	34.331397	4.7719

Discussion

- For the basic problem, all these methods perform similarly.
- It is hard to draw any conclusions with only 8 workers available (maximum on Socrates).
- Note that versions A and C are functionally the same, given how workers are assigned.
- Version B likely scales slightly worse than the others due to overhead associated with setting up the loops.
- It is likely that version D will scale better than the others.
- Version D requires that the full problem be stored in memory (as a 3D array) to obtain the eigenvalues, and this may require too much memory with the full problem.

Future Work

- Obviously the main extension is to use the full kernel $\mathscr{K}$ and solve the complete problem.
- Tests with the basic problem need to be done on a larger system to see which method actually scales the best.
- The convergence rates of the Chebyshev expansions also need to be examined.
- And finally, extension to 3-body problems.