University of Saskatchewan
Department of Mathematics

Numerical Analysis I
(MATH 211)
Instructor: Dr. Raymond J. Spiteri

MOCK Final Examination
8:30 a.m.– 9:45 a.m. Thursday, April 05, 2007

1. [**10 marks**]

(a) What is *roundoff error* and why does it arise?

(b) Explain the statement "Unit roundoff ($\epsilon_{\text{machine}}$) is not floating-point zero." What quantity would you use to define floating-point zero?

2. [**10 marks**]

(a) Define what is meant by a *permutation matrix*. Give the 3×3 permutation matrix $\mathbf{P}$ that would swap rows 2 and 3 of a 3×3 matrix $\mathbf{A}$. Would you ever actually multiply a matrix by $\mathbf{P}$ in order to swap its rows? Why or why not?

(b) Give 2 reasons why you should use the command

```
x = A \ b
```

and not

```
x = inv(A)*b
```

when solving a linear system $\mathbf{Ax} = \mathbf{b}$ in MATLAB.

3. [**10 marks**]

(a) Explain the connection between piecewise polynomial interpolation and Simpson's quadrature rule.

(b) Explain why a high-degree polynomial interpolant is not always more accurate than a low-degree piecewise polynomial interpolant. Where are you most likely to encounter the largest errors in high-degree polynomial interpolation?

4. [**30 marks**]

(a) A chemical reaction is taking place and the concentration of a certain ion at time t is given by $10e^{-3t} + 2e^{-5t}$. Set up an equation of the form $f(t) = 0$ whose solution yields the time t_* at which the concentration is half of its original value. Starting from the initial guess $t_0 = 0$, perform one Newton iteration to approximate t_*.

(b) Name two advantages and two disadvantages that the secant method has compared to Newton's method. Which is generally the more efficient method?

(c) Explain how the `zeroin` algorithm achieves the best of both worlds when it comes to being a foolproof and efficient algorithm.

5. **[20 marks]**

(a) *Weddle's rule* was derived from Simpson's rule by taking a weighted average of Simpson's rule with stepsize h on an interval $[a, b]$ and composite Simpson's rule with stepsize $h/2$. Derive Weddle's rule.

(b) Suppose you estimate the value of a definite integral with a quadrature rule with grid spacing h. Suppose you then compute another estimate with grid spacing $h/2$. What is the relationship between the size of the expected errors if the method has order p? By considering the work done versus the expected size of the error, explain why quadrature rules with higher order are more efficient than quadrature rules with lower order.

6. **[20 marks]**

(a) Convert the following third-order ODE to a system of first-order equations:

$$\dddot{x}(t) + \rho\dot{x}(t) = cx(t) + q(t).$$

Why must such a conversion be carried out in practice?

(b) Explain why modern software packages always equip initial-value problem solvers with the capability of taking steps with variable sizes.

(c) Given the following initial-value problem,

$$\begin{aligned} \dot{y_1} &= y_1 + y_2^2, & y_1(0) &= 1, \\ \dot{y_2} &= ty_1y_2, & y_2(0) &= 1, \end{aligned}$$

obtain an approximation to the solution at time $t = 0.3$ by taking 3 steps of Euler's method with stepsize $h = 0.1$.

(d) "High-order Runge–Kutta methods are more work per step than low-order Runge–Kutta methods." Explain why, despite this, it is advantageous to use high-order methods for some problems. What must be true about the problem and the accuracy requested for the advantages to be realized?